FINITE ELEMENTS SOFTWARE FOR ELECTROMAGNETICS APPLIED TO ELECTRICAL ENGINEERING TRAINING.

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Abstract

This paper reports an example of state-of-the-art technology applied to academic training of electrical engineers. An updated commercial Finite Element Method Software package is used to calculate electrical fields and power dissipation in a resistor with a complex geometry. These results are compared with theoretical solutions of a simplified model which can be obtained by the students with basic mathematics. Using a powerful tool to calculate electromagnetic fields very accurately let them to acknowledge the limitations of their theoretical solutions. The aim of this activity is the presentation of Electromagnetics theory in a more engaging manner, using a breakthrough technology as a bridge between the classical theoretical approach and industrial applications.

Index Terms. *Education, Electrical Engineering, Electromagnetics, Finite Elements Software (FES).*

1. Introduction.

Integration of basic sciences in Engineering raises learning difficulties associated with its theoretical foundation. Electromagnetics is a good example of a subject which has been found always difficult to learn by the student. A useful academic tool commonly use to make this subject more appeal to students is to show practical engineering examples related with the Maxwell equations (e.g. electrostatic and magnetic forces, induction heating, levitation phenomena, etc.) [1, 2]. A limitation of this approach is that the exact solution to the problem proposed can not be achieved using the basic mathematics available to the students in this academic level. To overcome this limitation, usually the real problem is simplified so it can be solved more easily. Although this method seems to be good enough, it does not satisfy whole students requirements. An intermediate stage between demanding high level of mathematics and software programming skills needed to solve those practical cases and the simplification proposed above, is to show the solutions achieved by means of numerical simulation, [3]. The high development of Finite Elements Method Software and its price reduction give the opportunity to use it not only for research but for learning purposes. [4, 5].

Analytical solutions of resistors with non-uniform distributions of current and power density are restricted to simplified geometries which are rough representations of the real model. These resistors present engineering problems as hot spots due to inhomogeneous power density distribution and its solution by means of FES is an example of an engineering study case which can be implemented in a basic electromagnetic course. We have solved a resistor with complex geometry with three different techniques. Two of them are analytical solutions of simplified models of the resistor which can be easily solved by the student. In the third one, the resistor was modelled and numerically solved using a commercial FES (Vector Fields). The post-processor attached to this software can show equipotencial surfaces, electric field lines, current density lines, power dissipation coloured map, etc. This graphic interface helps, for instance, to understand the effect of small curvature radius in the inhomogeneous distribution of Joule power dissipation. The numerical solution is compared with the results from the analytical methods.

In the last section of the paper, the principles outlined in the analysis of the former example will be used to study a real industrial resistor. A small automotive component, characterised by its complex geometrical shape, is analysed by means of FEM and real photographs of temperature distribution during lab tests are shown, [6].

2. A simplified case.

The chosen geometry for the resistor is an S-shape conductor sheet, with a negligible thickness, which can be considered to have only two dimensions. This piece with its electrical connections is shown in figure 1.

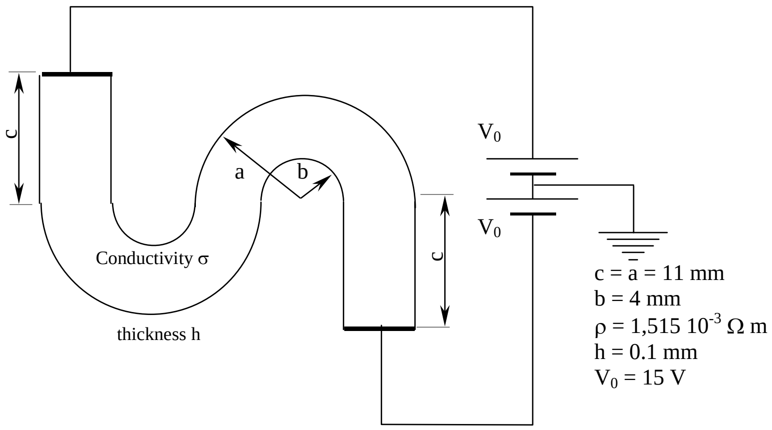

Figure 1. *Schematic of the resistor and its connections.*

When the current in a conductor is steady and there are not electromotive forces within the conductor, the value of the voltage V_o between the terminals of the conductor is proportional to the current I flowing through an straight section of the conductor. The coefficient of proportionality is called the resistance, R. If current and power density are considered constant along the volume of the material, e.g. a straight cylinder resistor with a well defined constant cross-section, the value of the coefficient R is given by the relation:

$$R = \rho \frac{l_{mean}}{S_{\text{section}}} \tag{1}$$

where the coefficient of proportionality ρ is the resistivity of the material, l_{mean} is the mean length of the conductor and S is the cross-section of the conductor. In a first approach, in the case where neither the cross-section nor the length is well defined, the student can roughly evaluate the resistance with the following expression:

$$R = \rho \frac{l_{mean}}{S_{\text{section}}} = \rho \frac{2\,\pi\,r_{mean} + 2\,c}{(a - b)\,h} = \rho \frac{2\,\pi\,\frac{a+b}{2} + 2\,c}{(a - b)\,h} \tag{2}$$

The second approach in order to increase accuracy for the calculations of the resistor is to split the piece in four independent resistors which can be connected in series. This model is depicted in figure 2. For the straight elements, R_1, current and power density are considered constant, and its resistance can be calculated with (1).

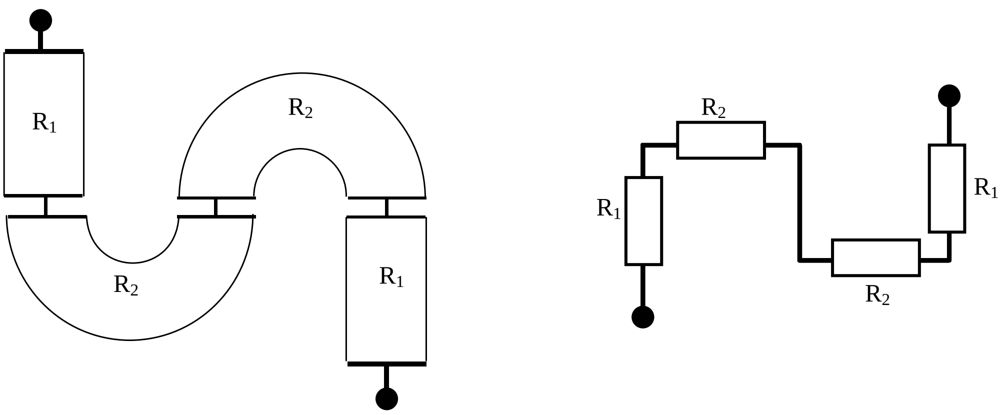

Figure 2. *Studied resistor divided into pieces and its equivalent circuit.*

The curved elements are more carefully analysed, taking into account the fact that the equipotencial lines are radial lines. Therefore the current density vector $\vec{J}$ has angular direction, perpendicular to the radial equipotential lines. The difficulty added to this geometry shall be easily overcomed by the student with the mathematical tools and theoretical descriptions included in a basic engineering course. Inaccuracy in the solution is due to the assumption that the junction surfaces of the four pieces are equipotentials. The equations for the resistance, current density and power density in the different zones are shown in table 1.

***Table 1**. Comparison of different values for each sections.*

	Straight zone	½ Circle
Resistance, R	$R = \rho \frac{l_{mean}}{S_{section}} = \rho \frac{c}{(a-b)\,h}$	$R = \rho \frac{\pi}{h \ln\left(a/b\right)}$
Current density, $\lvert\vec{J}\rvert$	$\lvert\vec{J}\rvert = \frac{I}{S_{section}} = \frac{I}{(a-b)\,h}$	$\lvert\vec{J}\rvert = \frac{I}{r\,h \ln\left(a/b\right)}$
Power density $p = \vec{J}\cdot\vec{E} = \rho\lvert\vec{J}\rvert^2$	$p = \rho \frac{I^2}{S_{section}{}^2} = \rho \frac{I^2}{(a-b)^2\,h^2}$	$p = \rho \frac{I^2}{r^2\,h^2\left[\ln\left(a/b\right)\right]^2}$

The last approach to solve the problem is to perform a numerical calculation of the electrical field by means of the finite element method. We have used for this purpose a commercial package (Opera 3D of Vector Fields). Figure 3 shows the contour map obtained with this software for the current density distribution in the conductor.

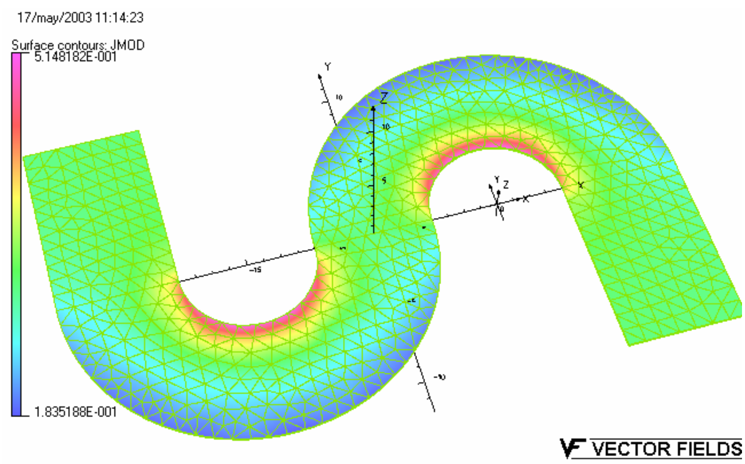

***Figure 3.** Current density distribution at the simplified piece.*

The numerical results are depicted in the figure with coloured surfaces, showing a uniform distribution of current in the straight element and non-uniform distribution of current in the curved elements.

Results achieved with the three methods implemented are shown in table 2. While the first approximation predicts a constant value for $\lvert\vec{J}\rvert$ and p in any point of the piece, the second one expects values which are dependant of the distance to the centre of the curved sections, coordinate r, as it can be seen in table 2. The results of the second method are closer to the ones obtained by FES, both for total values as resistance and for local values as $\lvert\vec{J}\rvert$ and p.

The error for the total resistance in the first method with respect to the FES value is 4,3% and 1,2% in the second one. At this point, is important to remark to the student that these calculations let us to choose the adequate approximation for the desired precision.

__Table 2__. Comparison between the different approximations.

	Resistance, R	*Current density,* $\left\lvert \vec{J} \right\rvert$	*Power density, p*
1st Approximation	149,6 Ω	286440 A/m^2	124,317 10^6 W/m^3
2nd Approximation	141,7 Ω	Straight section: 302390 A/m^2 Circular section: $\frac{1982,1}{r}$ A/m^2 (from 180000 to 495000 A/m^2)	Straight section: 138,547 10^6 W/m^3 Circular section: $\frac{5.95261}{r^2}$ W/m^3 (from 49,195 10^6 to 372,04 10^6 W/m^3)
Finite Element Method	143,4 Ω	Similar to data from 2nd approximation in the middle of each section, but noticeably different at the joint of sections.	Similar to data from 2nd approximation in the middle of each section, but noticeably different at the joint of sections.

Two phenomena observed only in the FES solution shall be underlined, as they give clues about the sources of inaccuracy in the two analytical methods. The transition between straight and curved zones, and the change of curvature radius in the junction of the curved pieces, implies distortion of $\vec{J}$ lines that disrupt the perpendicularity respect to the junctions. This deals to potential differences in the junction surface which were not considered in the analytical solution. The non-equipotentiality of the junction surface of the curved elements is depicted in figure 4.

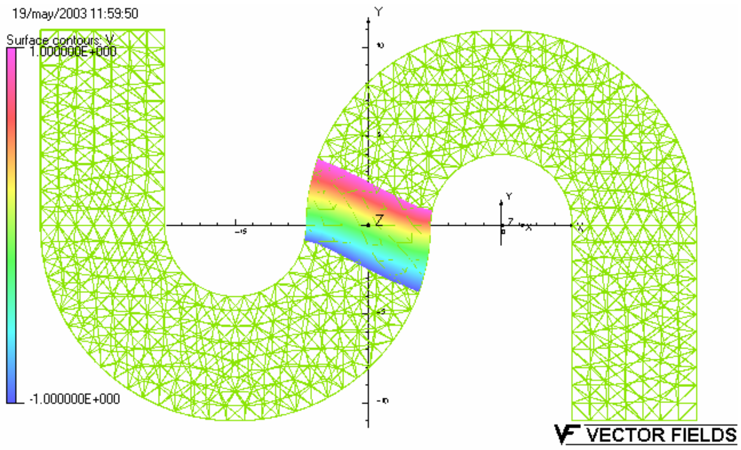

__Figure 4__. Graph of equipotential lines. (–1V < V < +1V).

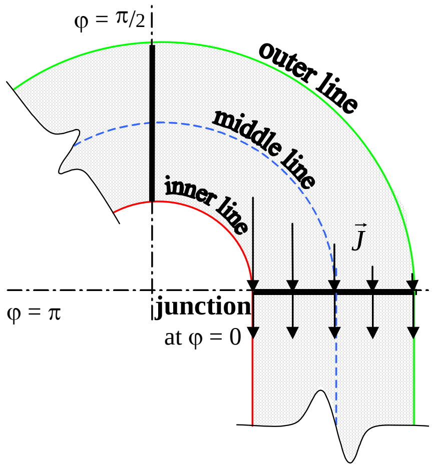

__Figure 5__. Violation of continuity law in the junction of straight and curved sections.

Violation of the continuity law for $\vec{J}$ in the junction of resistors R_1-R_2 of the second model is shown in figure 5. Continuity law for the electrical current can not be obeyed between the straight and curve pieces with the suppositions of that approximation, as it implies that $\vec{J}$ is constant in module for the flat zone and decreasing whit r in the curved one. In the FES solution, the equipotential lines are disturbed to achieve continuity in the junction, as it can be observed in figure 6.

The left side of figure 6 shows the variation of the module of the current density in the straight piece (resistance 1) obtained by FES for equidistant parallel paths, depicted in coloured lines and named outer, middle and inner lines. (see figure 5). It also includes the theoretical value obtained with the two approximations. The range of values in the module of the current density calculated with FES for the junction sections fulfils the continuity equation for current.

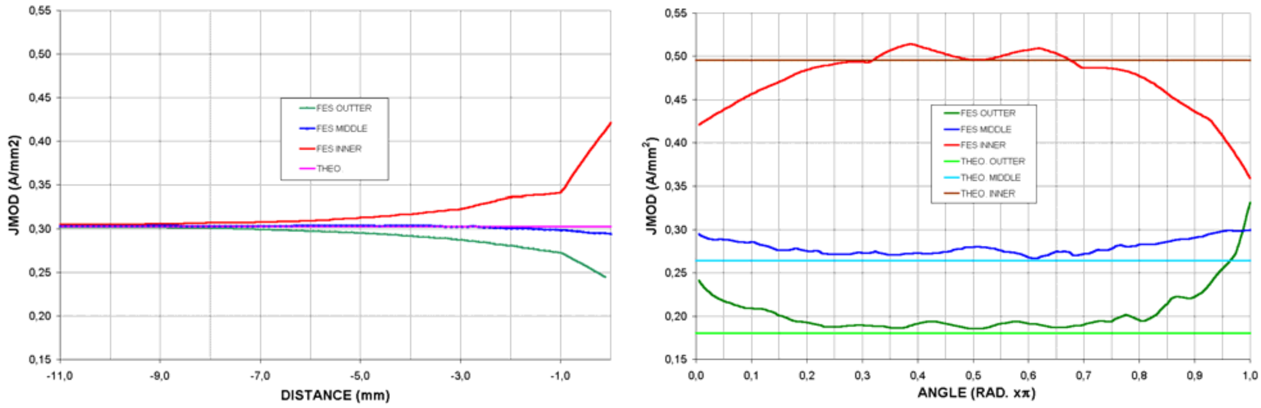

Figure 6. *Current density along the straight and curve sections for different parallel path. (comparison between second approach and FES solutions).*

In the right side of figure 6, the FES calculated values for the current density module along constant radius paths (inner, middle and outer line, see figure 5) of the curve piece (resistance 2) are depicted, and it includes the theoretical value obtained with the second approximation. It is clearly seen that the second approximation gives good values of the fields for all the sections which are far from those where the curvature radius change. In the junction sections where the radius of curvature changes rapidly with the angular position, only the FES is valid.

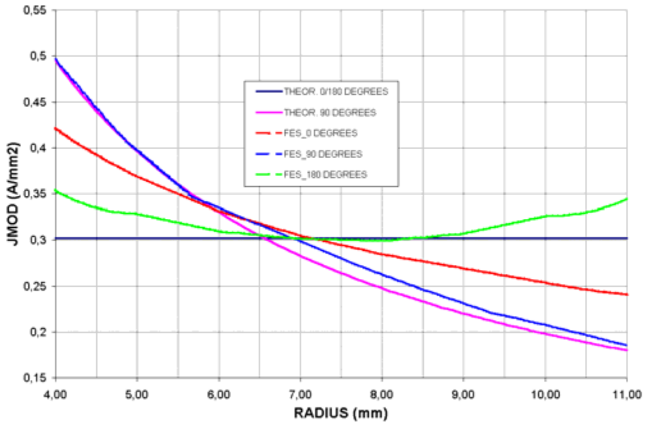

Figure 7*. Current density along a cross-section for different angular position. (analytical and FEM solutions).*

In figure 7, module of current density for three sections (0, $\pi/2$, π) of the curved piece (resistance 2) are located along the vertical axis. Radial distance of cross-sections are located along the horizontal axis. Values for sections 0 and π are compared with a straight line representing the values obtained with the second approximation. The same comparison is done for the section $\pi/2$, where the values obtained with the second approximation have a 1/r tendency.

It should have been interesting to get experimental data from a real model of the resistor. Only experimental results for the automotive piece shown in the next section can be presented. These results consist on colorimetric maps of temperature distribution. This experiment has the practical problem of high temperatures with its inherent security problems in student laboratories. To overcome these drawbacks, we are now working with thermo-chromic paints to design the experiment in the low temperature range.

3. Analysis of a real-case resistor.

In this section a complex, real resistor used in automotion application is analysed (see figure 8). The resistor is made from an Nicrom alloy (NiCr 6015), SIN DIN SEW 17470-17471. To calculate the electric field and the electrical resistance, we have followed the same procedure as for the former resistor. Three solutions for these magnitudes, two analytical and one numerical, are compared. In the second analysis, the piece is divided into two identical halves, each one containing six elemental resistors, connected in series. The results obtained with the three approximations are compared in table 3.

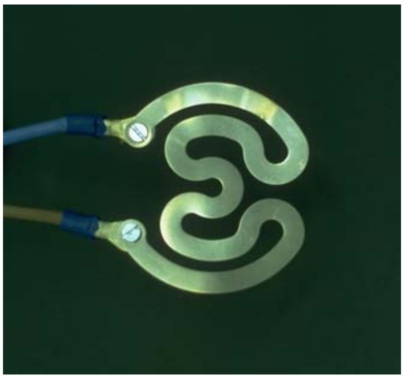

Figure 8. *Resistor use for the study case.*

Table 3. *Comparison of the resistance obtained in the three calculations.*

	1st Approx.	2nd Approx.	Finite Element Method
Resistance, R	57,3 Ω	55,6 Ω	53,2 Ω

To verify the solution obtained with FES, which are depicted in the coloured map of figure 9, a high electrical current was applied to the resistor to heat it below 973 K. Radiation at that temperature becomes visible, and colour in each point of the surface gives an indication of the different temperature. Heat transferred by conduction within the piece is small due to the negligible thickness of the piece. Therefore, temperature of the surface is expected to be proportional to the electrical power dissipation density, as confirmed with the laboratory photography (figure 10).

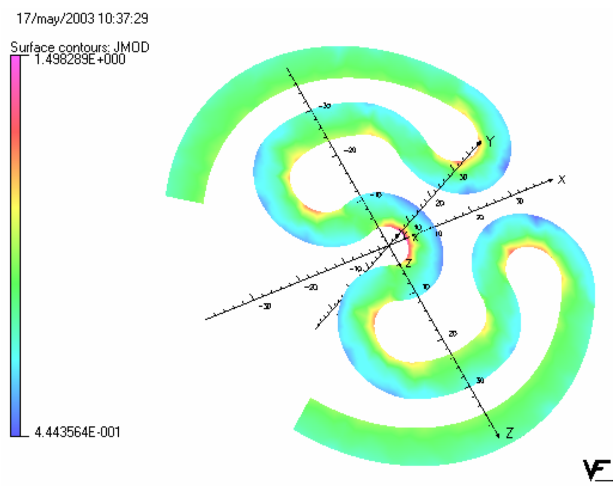

Figure 9. *Module of J contour map calculated by FES.*

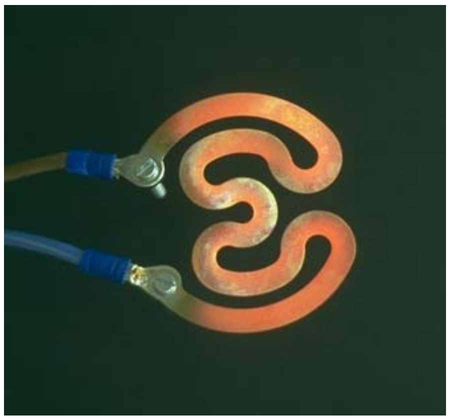

Figure 10. *Thermograph map for the resistance.*

It has been observed that the heat transference within the piece is small except in the connection terminals. The resistor is connected to the circuit through copper cables, of high thermal conductivity and appreciable section. In this case, the piece cools off through the terminals. For that reason, near the terminals the colour of the resistance in figure 10 does not correspond with the power density.

4. Conclusions.

The analytical resolution of an electromagnetic basic problem is, usually, a weak approximation to the real engineering problem. Therefore, it is convenient to develop realistic problem-solving techniques combining basic mathematics theory and separation into elemental straightforward cases, stressing the physics notions attached. The study-case solved in this paper shows that if the required precision is reduced, a basic approach to the problem is enough to reach an appropriate solution. However, if more accurate solution is required, improved methods should be used. Another remarkable aspect which can be interesting to present to the student is that the accuracy of the solutions depends on several factors. For example, in the resolution of the problem the temperature effect on the conductivity value is not considered, although it is not negligible if the piece is heated up to a temperature near the melting point. Also, if a high frequency AC-current flows through the resistor, skin effect can be also noticeable and the resistance value greatly increased.
To conclude, we consider interesting for the students to see, in a practical example, the need to use numerical models in many electromagnetic engineering problems.

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