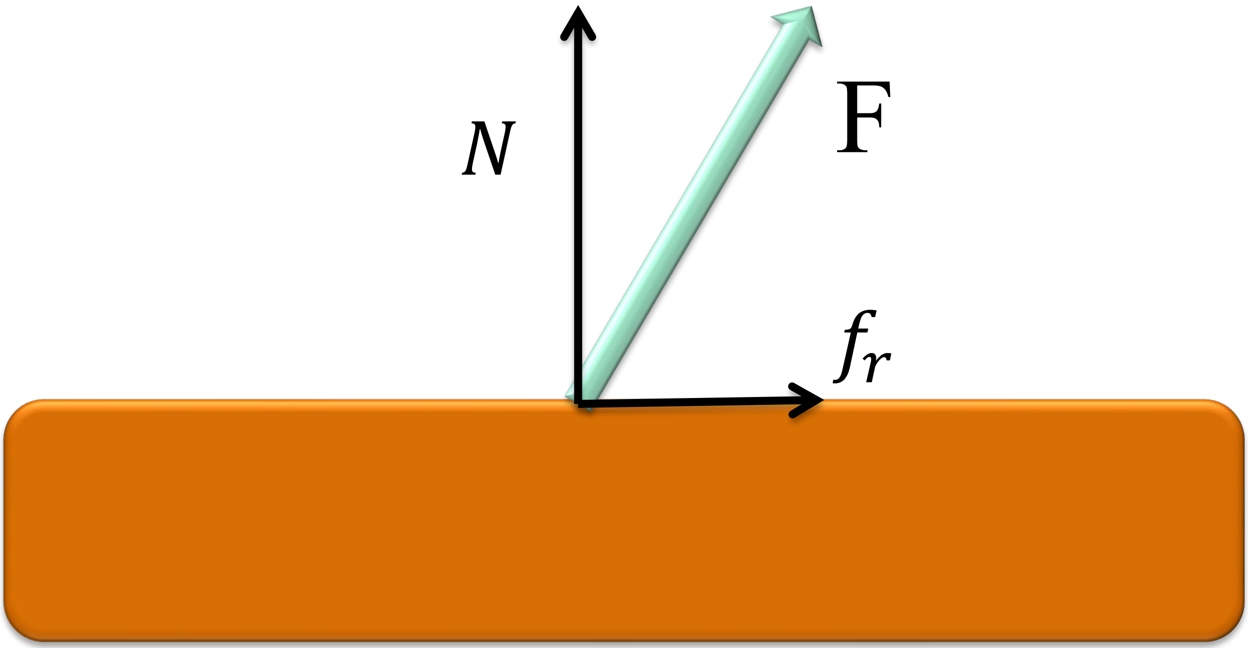
Leyes de Newton (el regreso)

Fuerzas especiales:

- Normal
- Rozamiento
- Muelle
- Centrípeta
  (Movimiento circular)
- Gravedad

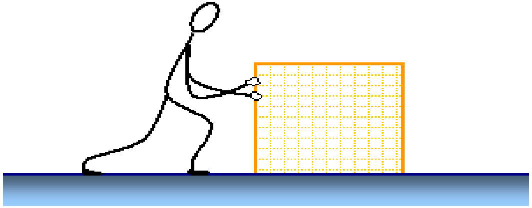
Ejemplos, ejemplos y ejemplos

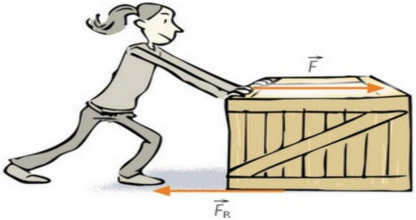
Una superficie ejerce fuerzas que descomponemos en:

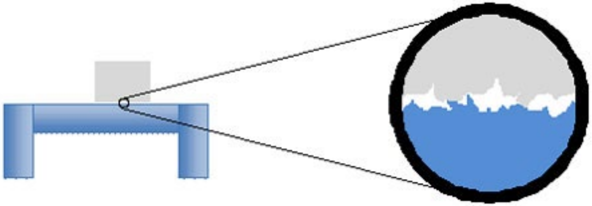


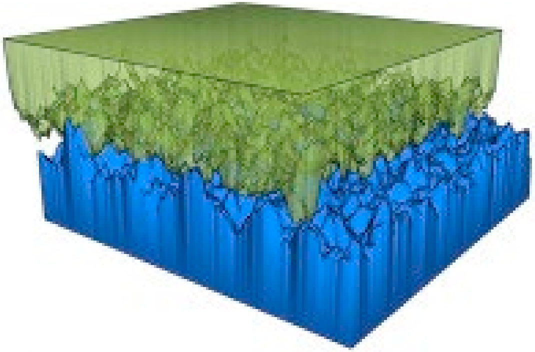
Si *fr=0, la única fuerza que puede ejercer la superficie es Normal a la superficie.*

Fuerza de rozamiento : f_r

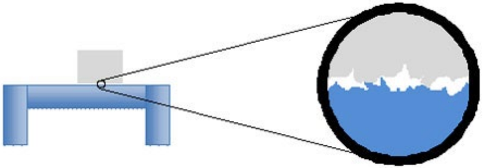


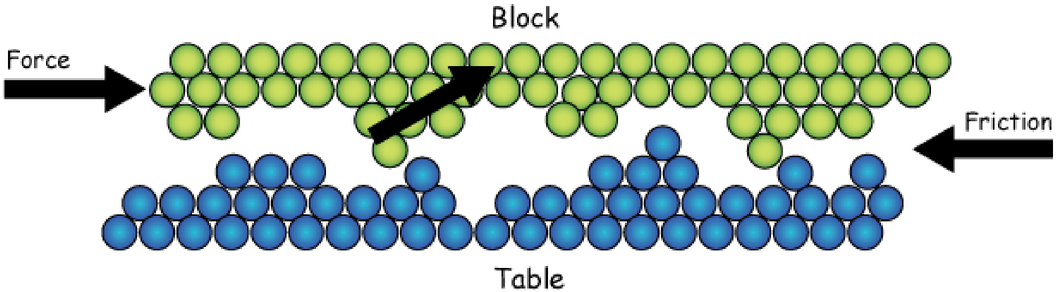


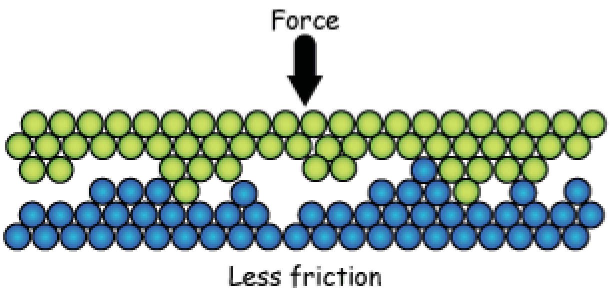


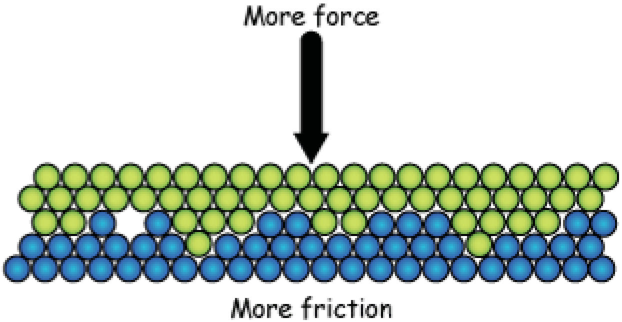


Fuerza de rozamiento :

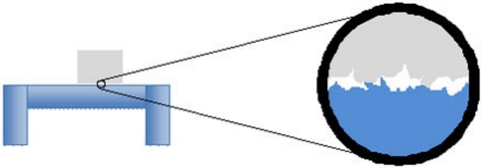


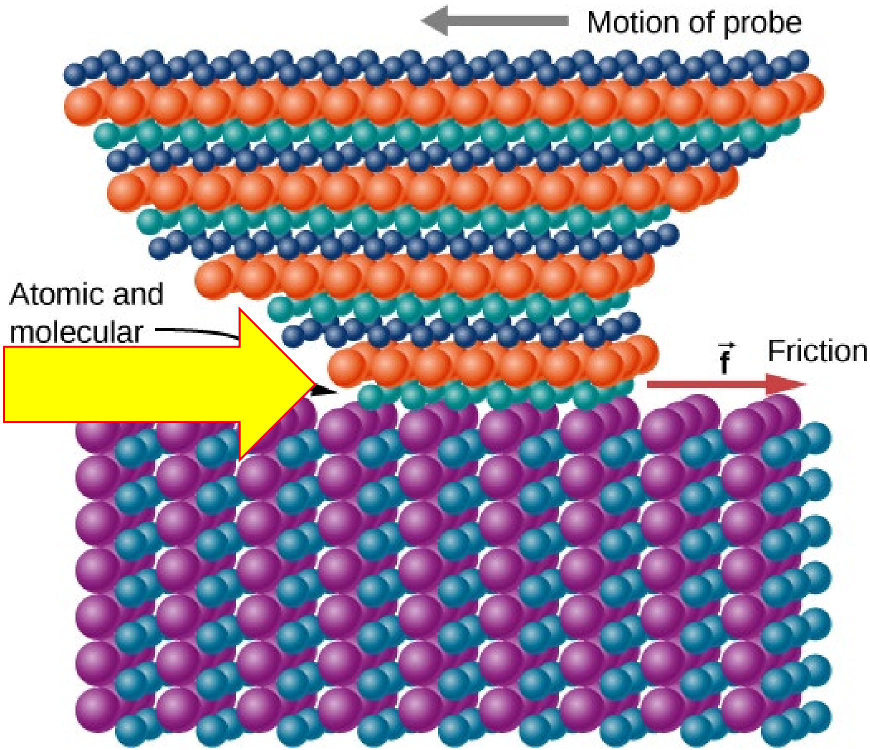






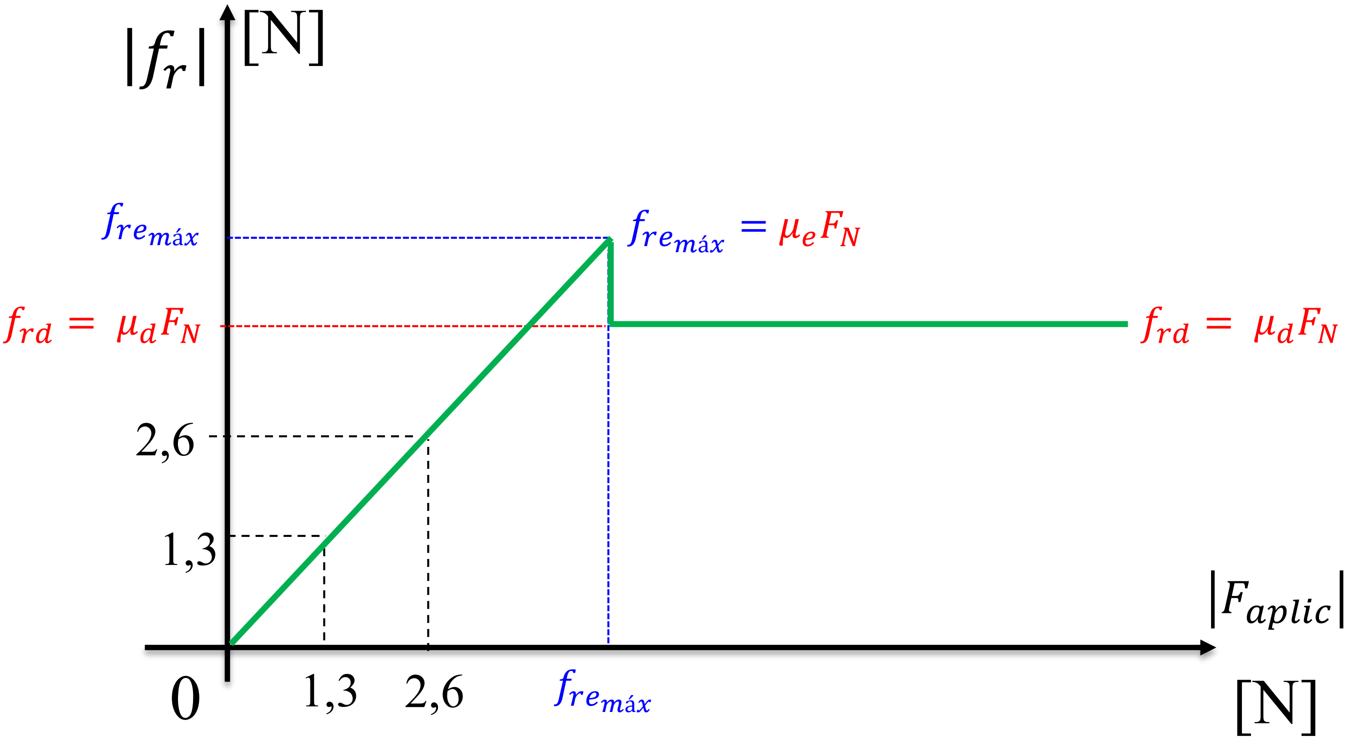
Fuerza de rozamiento :





Fuerza de rozamiento :

f_r



Fuerza de rozamiento :

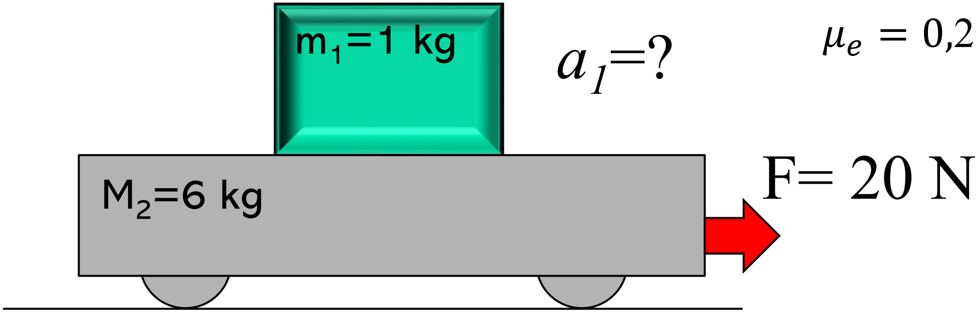
Las fuerzas de rozamiento dinámica máxima y estática máxima, vienen dadas por

$$fr_e = -F_{aplicada} \quad \text{si} \quad F_{aplicada} < \mu_e F_N$$

$$fr_e = \mu_e F_N \quad \text{Valor máximo}$$

$$fr_d = \mu_d F_N \quad \text{Valor único}$$

$$\mu_e > \mu_d \qquad \mu_e < 1 \, ; \, \mu_d < 1$$

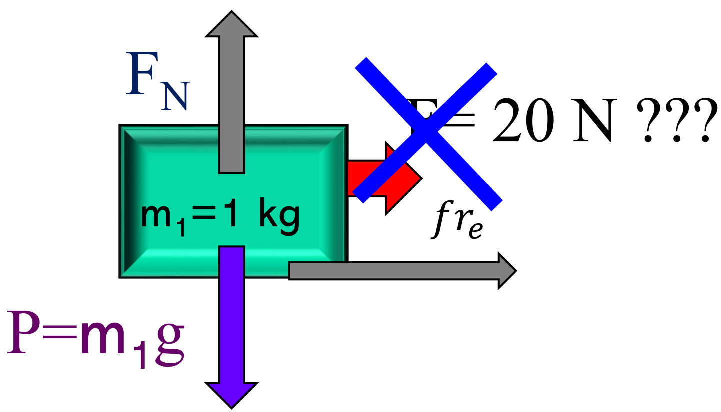


$\mu_d = 0{,}1$

$\mu_e = 0{,}2$

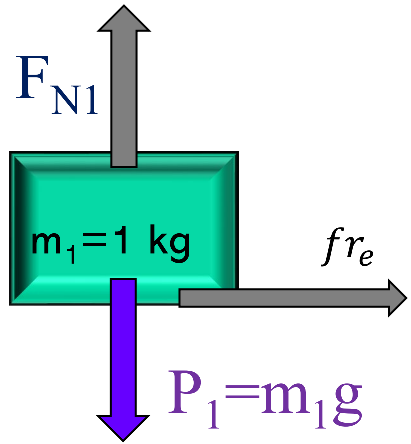
Si quiero conocer a_1, mi sistema **deberá ser** m$_1$

$$\sum_{i=1}^{N} \vec{F}_{externas} = m\vec{a}$$



Sistema 1

$$\sum_{i=1}^{N} \vec{F}_{ext} = m_1 \vec{a}_1$$



$$\sum_{i=1}^{N} F_x = m_1 a_{1x}$$

$$\sum_{i=1}^{N} F_y = m_1 a_{1y} \quad = 0$$

$$\sum_{i=1}^{N} F_x = fr_e = m_1 a_{1x} \quad \Rightarrow \quad fr_e = m_1 a_{1x}$$

$$\sum_{i=1}^{N} F_y = F_{N1} - P_1 = 0 \quad \Rightarrow \quad F_{N1} = P_1 = m_1 g$$

$$F_{N1} = P_1 = 1kg \times 10\frac{m}{s^2} = 10\ N$$

$F_N = P_1 = 10\ N$ ⇒ $fr_e^{max} = \mu_e\ F_N = m_1 a_{1x}$

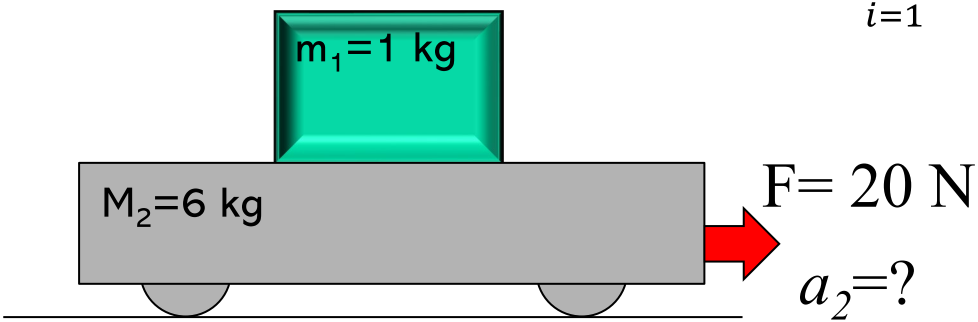
$$fr_e^{max} = \mu_e\ F_N = 0{,}2 \times 10N = 2\ N$$

⇓

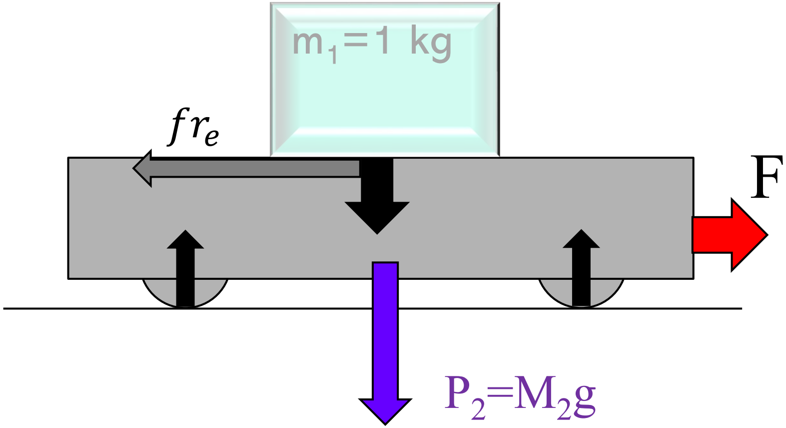
$$a_{1x} = \frac{2N}{1\ kg} = 2\ m/s^2$$

Como *fr* es la fuerza de rozamiento *máxima*, la aceleración *a* *será la máxima aceleración posible de la masa* m_1.

$$\sum_{i=1}^{N} \vec{F}_{externas} = m\vec{a}$$

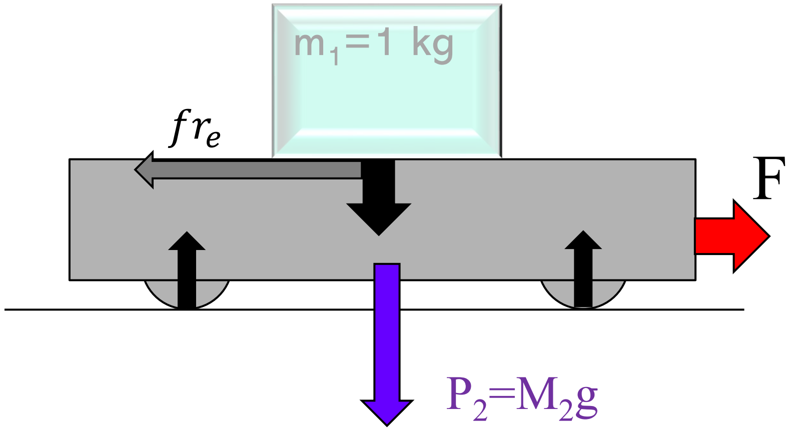


Si quiero conocer a_2, mi sistema **deberá ser** M_2



Sistema 2

$$\sum_{i=1}^{N} \vec{F}_{ext} = M_2 \vec{a}_2$$



$$\sum_{i=1}^{N} F_x = M_2 a_{2x}$$

$$\sum_{i=1}^{N} F_y = M_2 a_{2y} \quad = 0$$

$$\sum_{i=1}^{N} F_x = F - fr_e = M_2 a_{2x} \;\Rightarrow\; F - fr_e = M_2 a_{2x} \;\Rightarrow\; \frac{(F - fr_e)}{M_2} = a_{2x}$$

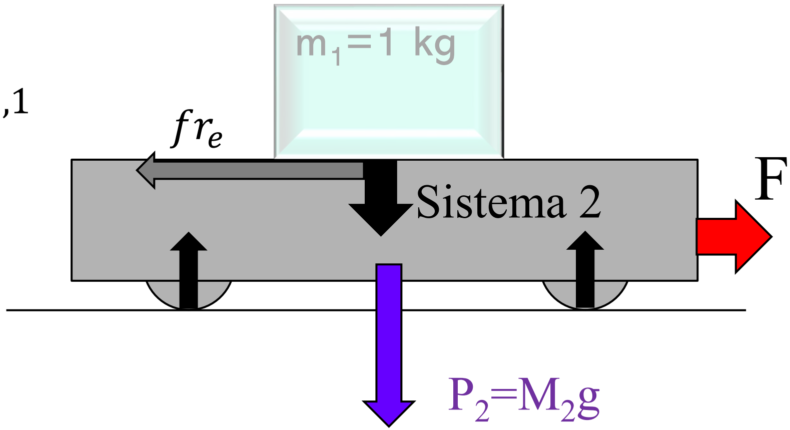
$$\frac{(F - fr_e)}{M_2} = a_{2x} = \frac{20 - 2}{6} = 3\ m/s^2$$

$$a_{1x} = \frac{2N}{1\ kg} = 2\ m/s^2$$

$$a_{2x} > a_{1x} \quad !!!$$

Entonces debo usar

$$fr_d \quad con \;\; \mu_d = 0{,}1$$



$$\sum_{i=1}^{N} \vec{F}_{ext} = M_2 \vec{a}$$

$$\sum_{i=1}^{N} F_x = M_2 a_{2x}$$

$$\sum_{i=1}^{N} F_y = M_2 a_{2y} \;\; = 0$$

$$\sum_{i=1}^{N} F_x = F - fr_d = M_2 a_{2x} \;\Rightarrow\; F - fr_d = M_2 a_{2x} \;\Rightarrow\; \frac{(F - fr_d)}{M_2} = a_{2x}$$

$$\frac{(F - fr_d)}{M_2} = a_{2x} = \frac{20 - 1}{6} = 3{,}166\, m/s^2$$

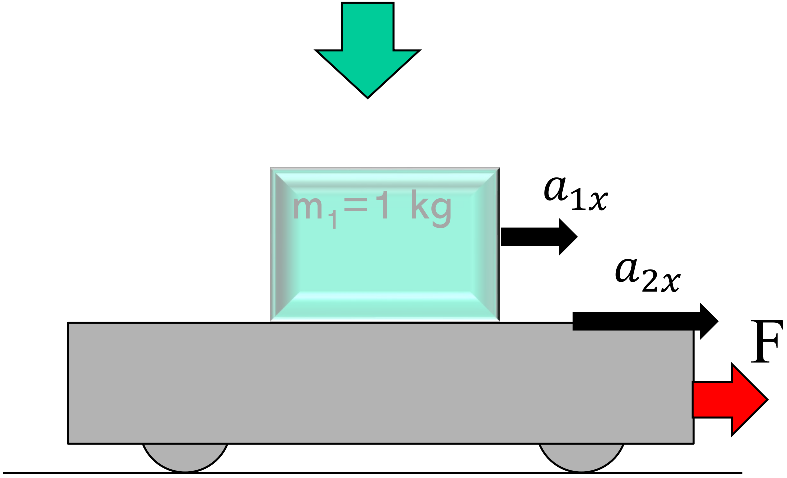
$$a_{1x} = \frac{2N}{1\,kg} = 2\, m/s^2$$

Entonces la situacion es que las superficies deslizan una sobre otra, y por tanto

$$fr = fr_d$$

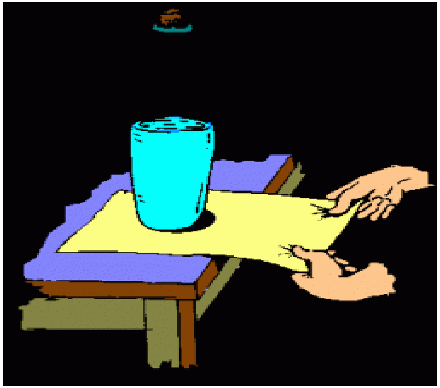
Y obtenemos

$$a_{1x} = 1\, m/s^2 \qquad a_{2x} = 3{,}16\, m/s^2$$



Rozamiento I:

Qué pasa aquí??



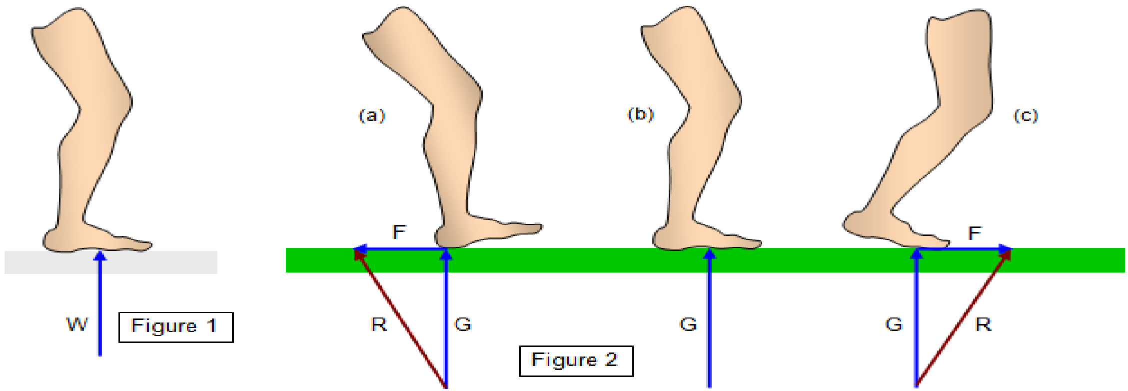
Tips:

- ✓ Inercia: La tendencia de un objeto a resistir ***aceleraciones.***
- ✓ ***Fr dinámica vs. estática.***
- ✓ $\boldsymbol{F} = \frac{\Delta\vec{p}}{\Delta t}$

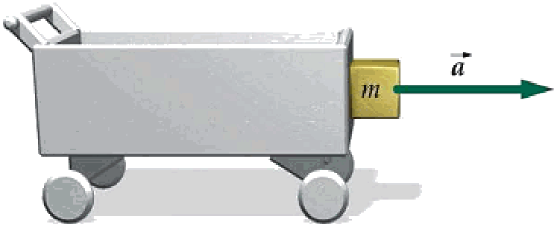
https://youtu.be/PLpav01H_60

Trabajo tutelado ??

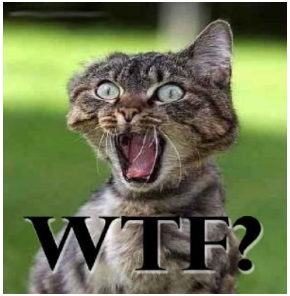
Rozamiento II:



Fuerza Normal y fuerza de rozamiento :



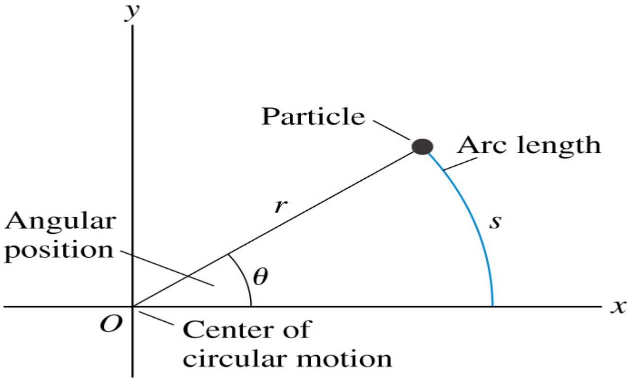
$$f_r \ , \mathrm{F_N}$$



Radians

$$\theta(radians) \equiv \frac{s}{r}$$

$$s = r\theta$$



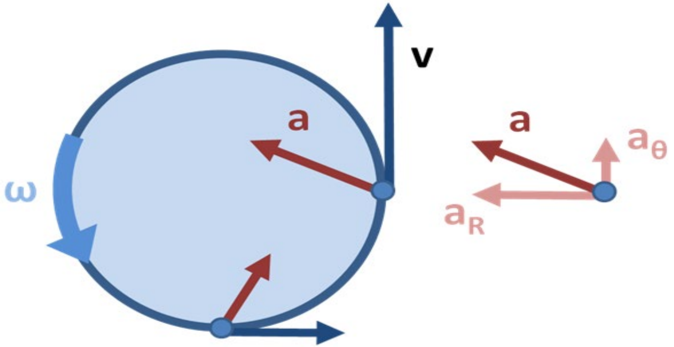
For a full circle.

$$\theta_{fullcircle} \equiv \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \quad \text{rad}$$

$$1 \quad rev = 360^0 = 2\pi \quad \text{rad}$$

$$1 \quad \text{rad} = 1 \quad \text{rad}\,\frac{360^0}{2\pi \quad \text{rad}}$$

De cinemática…

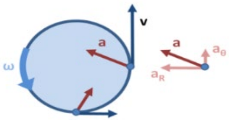


$$\vec{a} = \frac{d\vec{\mathbf{v}}}{dt} = \underbrace{\frac{d\mathrm{v}}{dt}}_{a_t}\breve{\boldsymbol{u}}_t + \underbrace{\frac{\mathrm{v}^2}{R}}_{a_n}\breve{\boldsymbol{u}}_n$$

$$\Rightarrow \begin{cases} a_t = \alpha R \\ a_n = \dfrac{\mathrm{v}^2}{R} \end{cases}$$

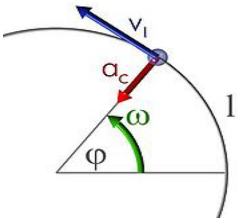
http://www.sc.ehu.es/sbweb/fisica_//cinematica/curvilineo/curvilineo/curvilineo1.html

Remembering…



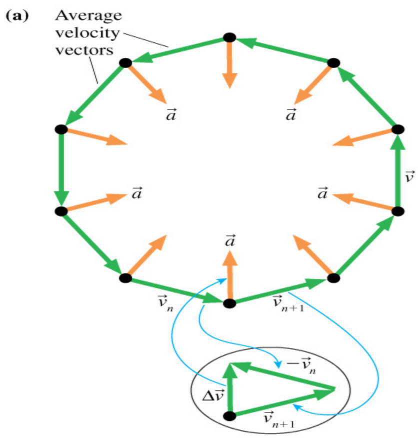
$$\vec{a} = \frac{dv}{dt}\breve{u}_t + \frac{v^2}{R}\breve{u}_n$$

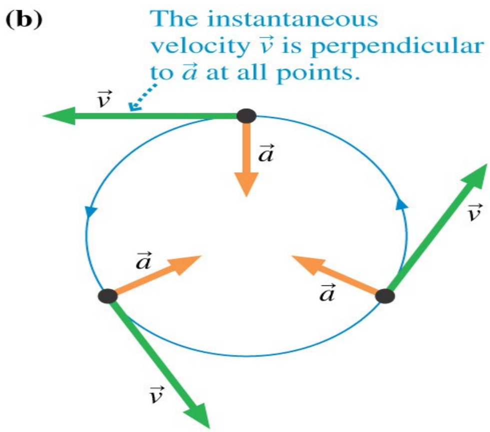
$$a_t = \frac{F_t}{m} \qquad a_R = \frac{F_R}{m}$$



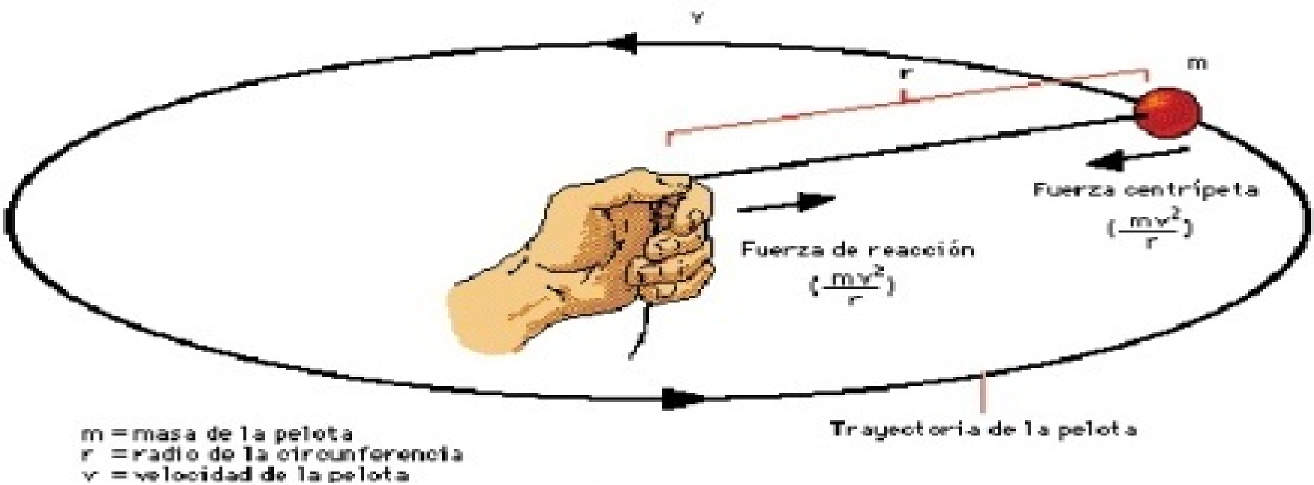
http://enc.edu/~john.u.free/PY%20201%202011-12/Frameset.htm

Entonces hay una aceleración!



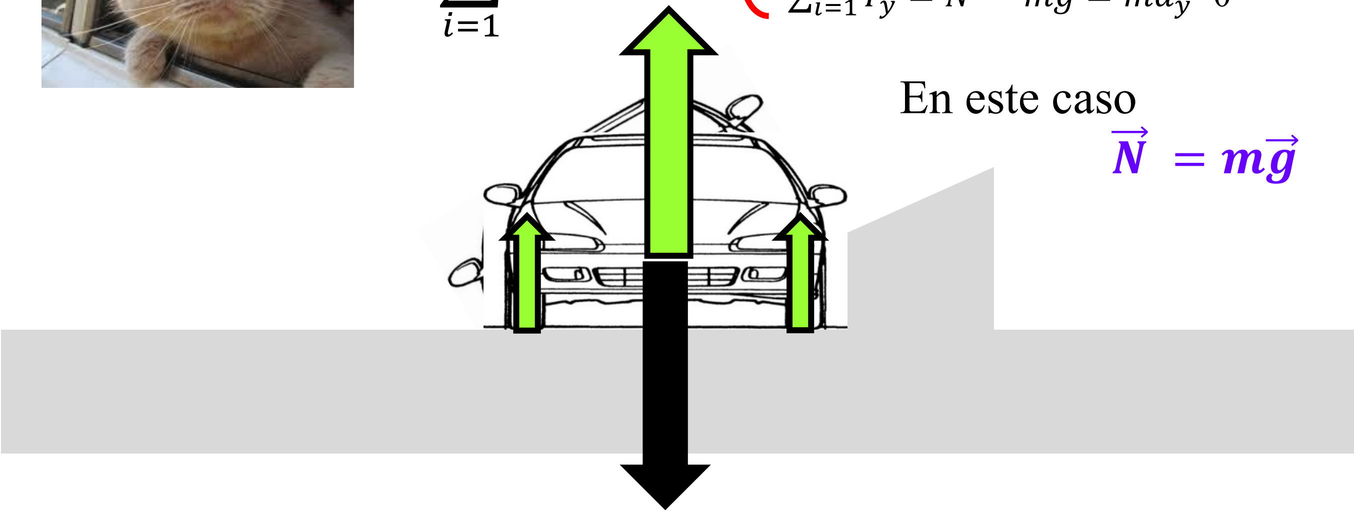


http://enc.edu/~john.u.free/PY%20201%202011-12/Frameset.htm



$$F_c = ma_c = m\frac{\mathrm{v}^2}{R} \qquad F_t = 0$$

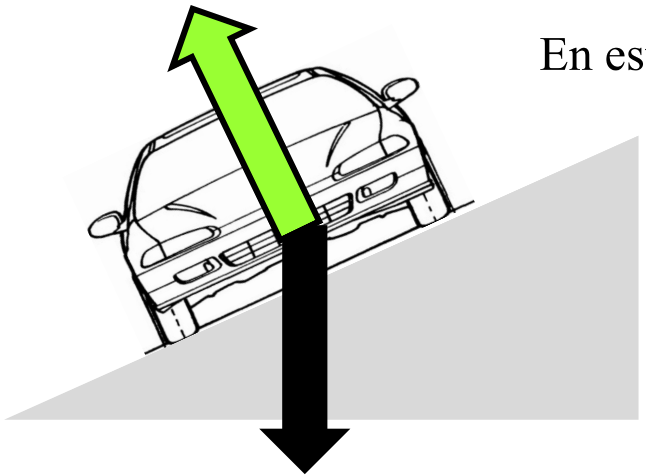
$$\sum_{i=1}^{N} \overrightarrow{F_i} = m\vec{a} \quad \begin{cases} \sum_{i=1}^{N} F_x = ma_x = 0 \\ \sum_{i=1}^{N} F_y = N - mg = ma_y = 0 \end{cases}$$



En este caso

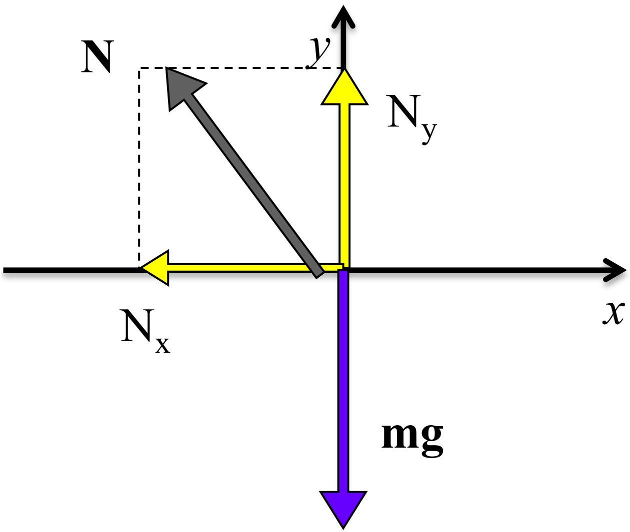
$$\vec{N} = m\vec{g}$$

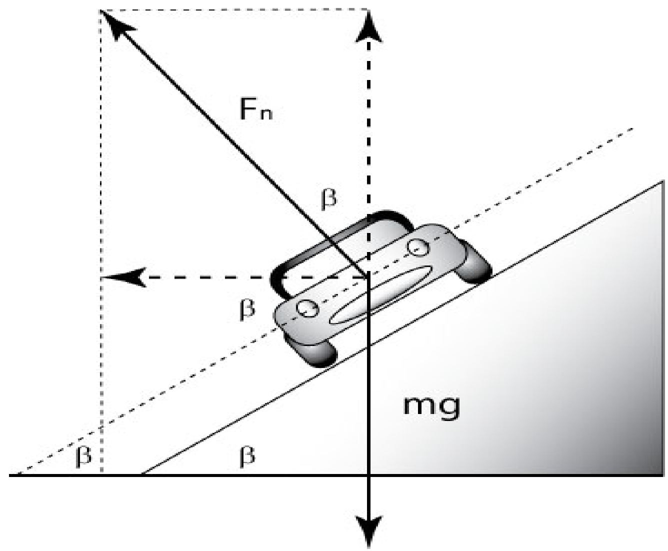
$$\sum_{i=1}^{N} \overrightarrow{F_i} = m\vec{a} \quad \begin{cases} \sum_{i=1}^{N} F_x = ? \\ \sum_{i=1}^{N} F_y = ? \end{cases}$$



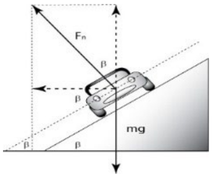
En este caso

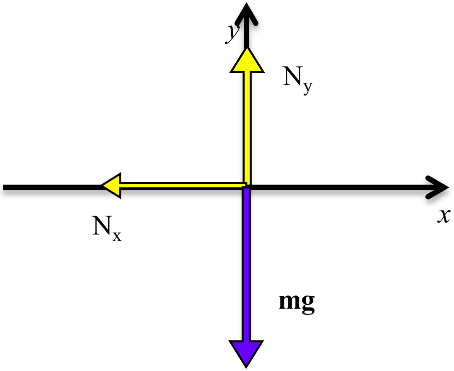
$$\vec{N} = m\vec{g}$$


N
y
Ny
Nx
x
mg


Fn
β
β
β
β
mg

$$\sum_{i=1}^{N} \overrightarrow{F_i} = m\vec{a} \quad \begin{cases} \sum_{i=1}^{N} F_x = ma_x = ma_c = \\ \sum_{i=1}^{N} F_y = N - mg = ma_y = 0 \end{cases}$$



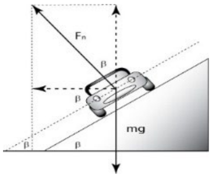


$$\sum_{i=1}^{N} F_x = N_x = N\,sen\beta = ma_n$$

$$\sum_{i=1}^{N} F_y = N_y - mg = N cos\beta = ma_t$$

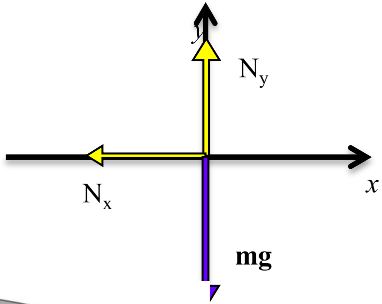
$$N\,sen\beta = \ ma_n = m\frac{\mathrm{v}^2}{R}$$

$$Ncos\beta - mg = ma_t = 0$$



En este caso

$$\overrightarrow{\boldsymbol{N}} \neq \boldsymbol{m}\overrightarrow{\boldsymbol{g}}$$

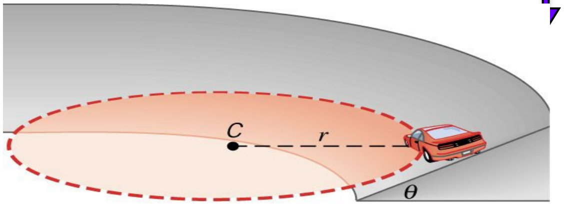


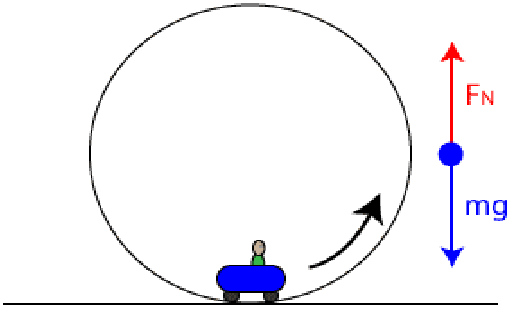
Pero además

$$N\,sen\beta = m\frac{\mathrm{v}^2}{R}$$

$$Ncos\beta = mg$$

$$tg\beta = \frac{\mathrm{v}^2}{Rg}$$





FN
mg

Leyes de Newton (el regreso)

Fuerzas especiales:

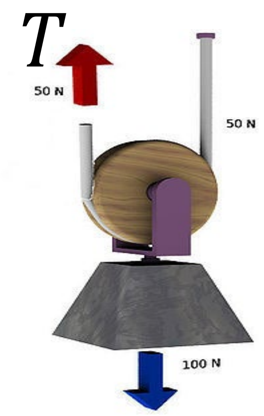
- Normal
- Rozamiento
- Muelle
- Centrípeta
  (Movimiento circular)
- Gravedad

Ejemplos, ejemplos y ejemplos

Particularidades de la fuerza T (tension en una cuerda):

La cuerda ideal que consideramos:

- Es inextensible pero compresible (sólo 'tira').
- No tiene masa (por ahora).



$$\sum_{i=1}^{N} F_y = 2T - T - T = ma_y = 0$$

Tension en una cuerda: T

Sistema 1

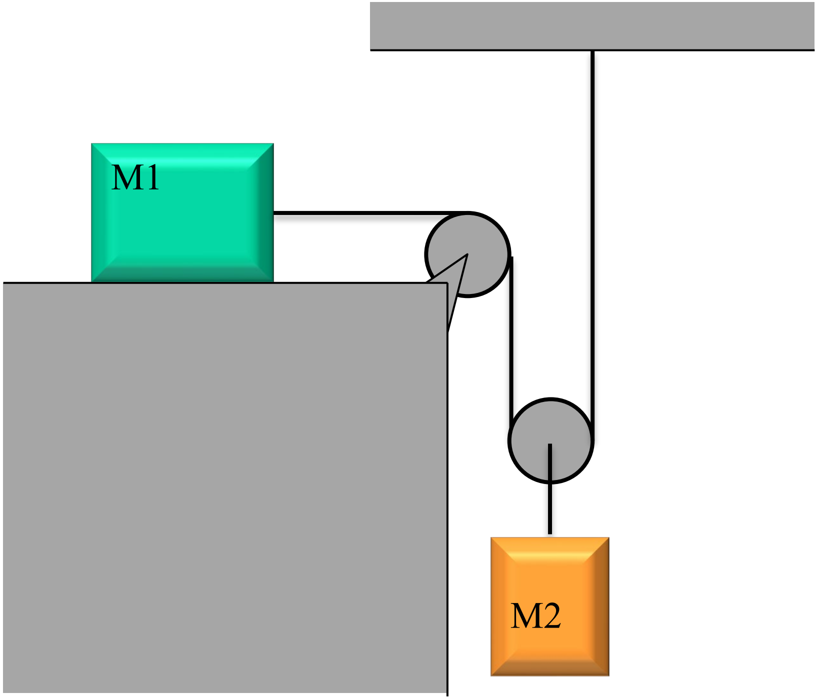
$$\sum_{i=1}^{N} F_x = T - fr = M_1 a_{1x}$$

$$\sum_{i=1}^{N} F_y = N - M_1 g = 0$$

Sistema 2

$\sum_{i=1}^{N} F_x = 0$

$$\sum_{i=1}^{N} F_y = 2T - M_2 g = M_2 a_{2y}$$



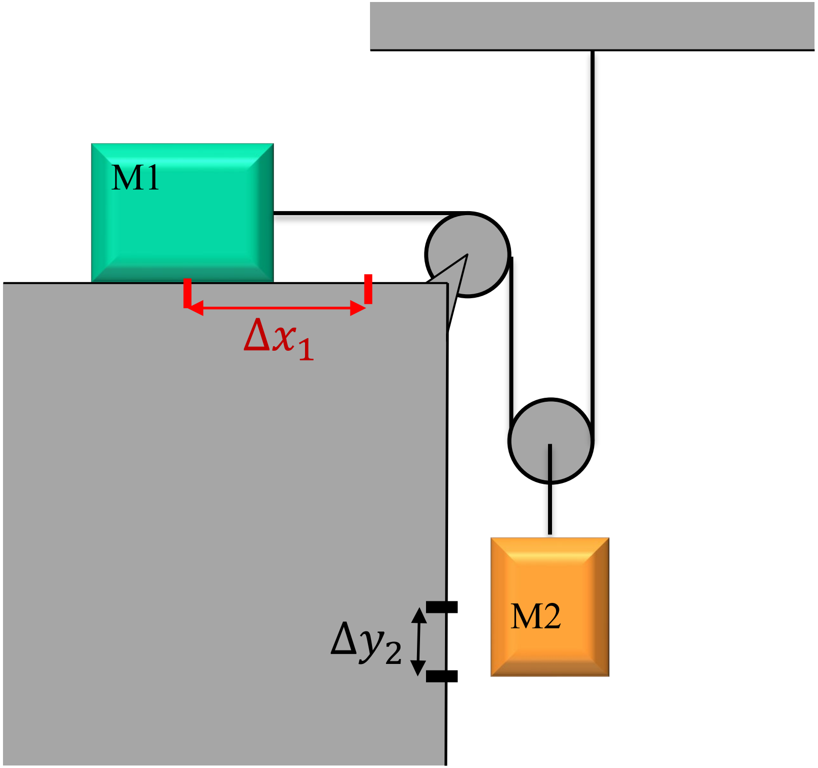
Tension en una cuerda: T

$$\Delta x_1 = 2\Delta y_2$$

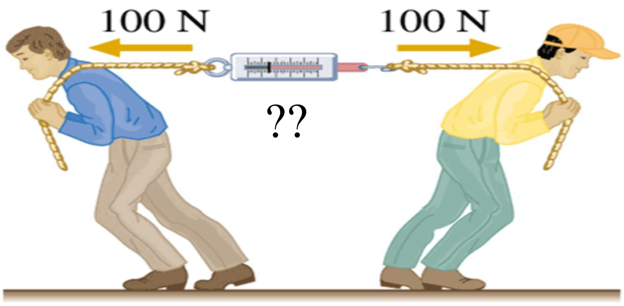
$$\frac{\Delta x_1}{\Delta t} = 2\frac{\Delta y_2}{\Delta t}$$

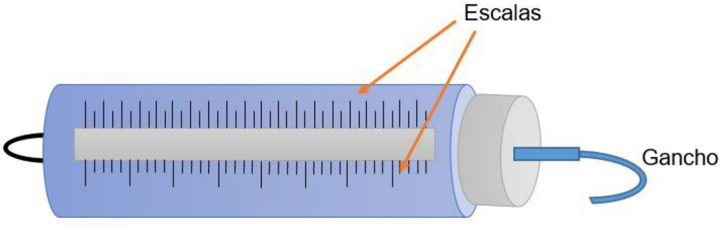
$$v_{1x} = 2v_{2y}$$

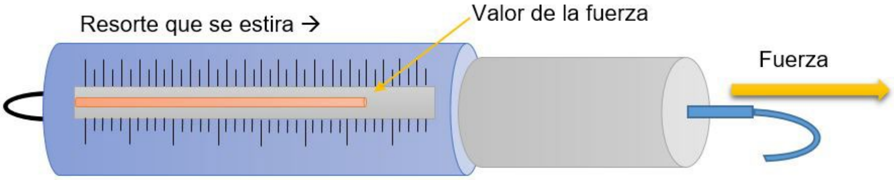
$$a_{1x} = 2a_{2y}$$



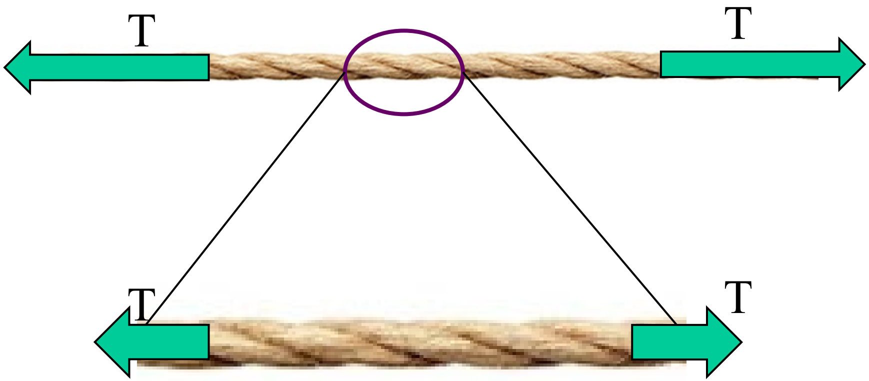
Particularidades de la fuerza T (tension en una cuerda): T







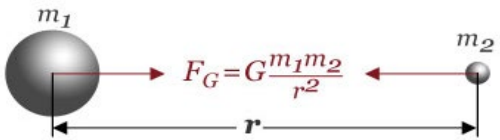
Particularidades de la fuerza T (tension en una cuerda): T



Fuerzas Especiales (Cont.)

- ✓ Fuerza gravitacional. $\vec{F}(\vec{r})$
- ✓ Fuerza viscosa. $\vec{\mathrm{F}}(\vec{\mathrm{v}})$
- ✓ Fuerza elástica. $\vec{\mathrm{F}}(\mathrm{x})$

$$F_G = G\frac{M.m}{R^2}$$



G = 6,6738x10^{-11} N (m/kg)^2

$$F_G = 6,67 \times 10^{-11} N \left(\frac{m}{kg}\right)^2 \times \frac{1\,kg \ \times 1\,kg}{(1m)^2} = 6,67 \times 10^{-11} N$$

$$F_G(R) = GMm\left[\frac{1}{(R)^2}\right]$$

$$\frac{1}{(R+h)^2} \approx \frac{1}{(R)^2}$$

$R = 6371000$ m

$(R+h) = 6371010$ m

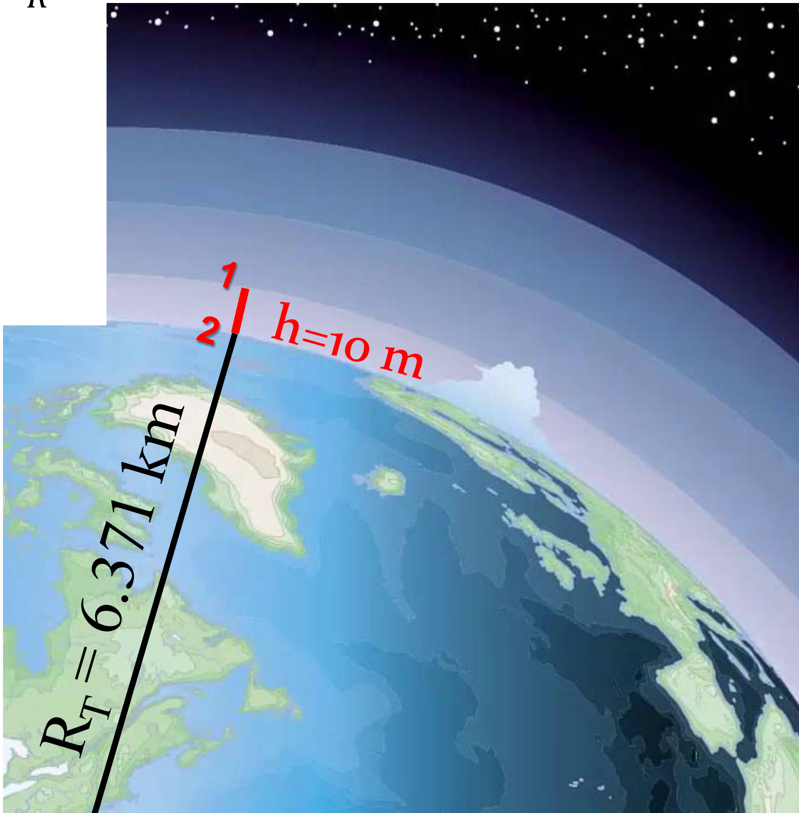
$$F_G(R+h) = GMm\left[\frac{1}{(R+h)^2}\right] \approx GMm\frac{1}{R^2}$$

$$F_G(R+h) \approx F_G(R) = GMm\frac{1}{R^2}$$

$$F_G = G\frac{M}{R^2}m \approx g\,m$$

$Si\ R \gg h$

$$F_G \approx mg$$



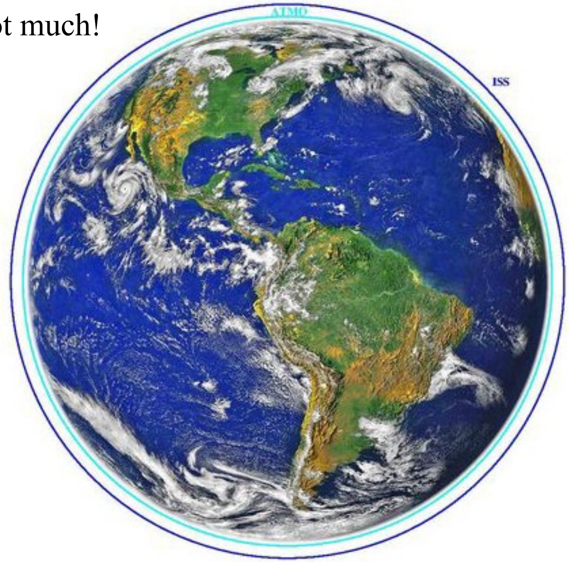
$$F_G = G\frac{M}{R^2}m = g\,m$$

$$g = \frac{6,67x10^{-11} \times 5,98x10^{24}}{(6,37x10^{6})^{2}}\ (m/s^2) \cong \ 9,8\,m/s^2$$

$F_G = F(r)$ o bien F = mg

https://phet.colorado.edu/en/simulation/gravity-and-orbits

How far do astronauts go? Not much!



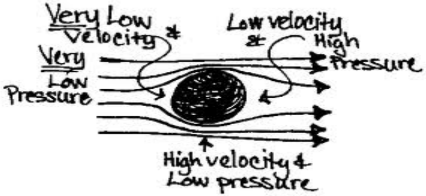
Hector Socas Navarro

¿Cuán lejos van los astronautas? ¿A qué altura está "el espacio exterior"? A veces no somos conscientes pero la órbita baja es muy, muy cerquita de la Tierra. He preparado este esquemita en el que vemos la línea de Kármán (referencia del fin de la atmósfera) y la altitud de la Estación Espacial Internacional (ISS).
¿Y la gravedad? Pues la gravedad de la Tierra a la altura a que está la ISS no cambia mucho. Una masa de 1kg pesaría como si tuviera 900g a esa altura.

El porqué parece no haber gravedad en la ISS es por el hecho de que está en caída libre. Formalmente una órbita es equivalente a una caída pero que no termina nunca. Un objeto en caída libre no experimenta efectos de gravedad (esto tiene que ver con el principio de equivalencia). En realidad es más preciso decir microgravedad por las pequeñas aceleraciones, mareas, etc. También se puede lograr microgravedad dejando caer un avión en caída libre. Pero claro, en un avión tienes limitado cuánto tiempo puedes estar cayendo. No solo porque te topas con el suelo (ahí se rompe la magia de la gravedad cero... bueno, y todo) sino por el roce con el aire.

Fuerzas Especiales (Cont.)

- ✓ Fuerza viscosa.



TERMINAL VELOCITY

$$F_v = -bv$$

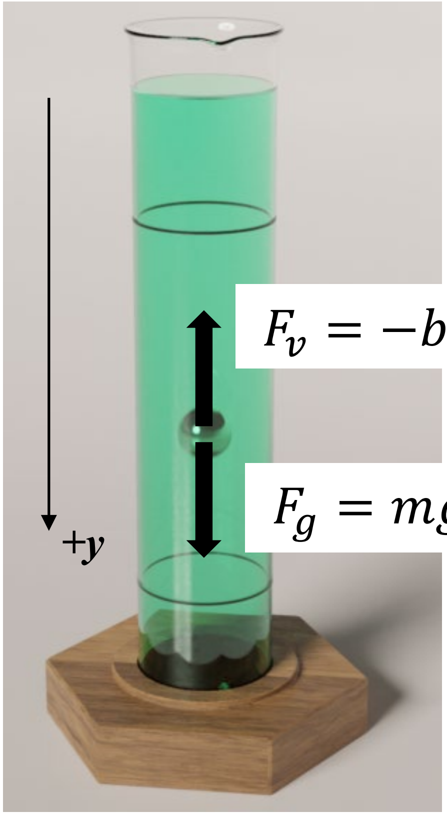
$$\sum_{i=1}^{N} F_i = mg - bv = ma$$

$$mg - bv = ma$$

$$mg - bv = m\frac{dv}{dt}$$

$$m\frac{dv}{dt} + bv - mg = 0$$

$$\frac{dv}{dt} + \frac{b}{m}v - g = 0$$



TERMINAL VELOCITY

$$\frac{b}{m} = \beta$$

$$\frac{dv}{dt} + \frac{b}{m}v - g = 0$$

$$\frac{dv}{dt} + \beta v - g = 0$$

$$\mu = \beta v - g$$

$$\frac{d\mu}{dt} = \beta \frac{dv}{dt}$$

$$\frac{1}{\beta}\frac{d\mu}{dt} = \frac{dv}{dt}$$

$$\frac{1}{\beta}\frac{d\mu}{dt} + \mu = 0$$

$$\frac{d\mu}{dt} + \beta\mu = 0$$

$$\mu(t) = Ae^{-\beta t}$$

Vamos a demostrar que $\mu(t) = e^{-\beta t}$ Es una solución de la ecuación

$$\frac{d\mu}{dt} + \beta\mu = 0$$

$$\frac{1}{\beta}\frac{d\mu}{dt} + \mu = 0$$

$$\mu(t) = Ae^{-\beta t}$$

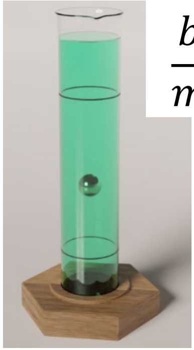
$$\frac{d\mu}{dt} = -\beta A e^{-\beta t}$$

$$\frac{A}{\beta}\left(-\beta e^{-\beta t}\right) + A\left(e^{-\beta t}\right) = 0$$

$$-Ae^{-\beta t} + Ae^{-\beta t} = 0$$

$\mu(t) = Ae^{-\beta t}$ es solución de la ecuación $\frac{d\mu}{dt} + \beta\mu = 0$

TERMINAL VELOCITY



$$\frac{b}{m} = \beta$$

$$\mu(t) = Ae^{-\beta t} \qquad \mu = \beta v - g$$

$$v(t) = \frac{1}{\beta}[\mu(t) + g]$$

$$v(t) = \frac{1}{\beta}\left(Ae^{-\beta t} + g\right) = \frac{m}{b}\left(g + Ae^{-\beta t}\right)$$

$$v(t = \infty) = v_L = \frac{mg}{b} \Rightarrow v_L = \frac{mg}{b}$$

$$v(t = 0) = 0 \Rightarrow A = -g$$

$$v(t) = v_L\left(1 - e^{-\beta t}\right)$$

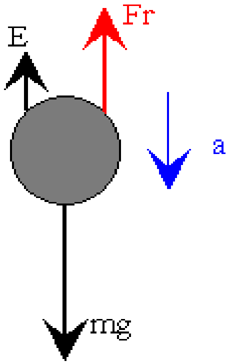
TERMINAL VELOCITY

$$\sum_{i=1}^{N} F_i = mg - E - F_S = ma$$

$$mg - m_l g - F_S = ma$$

$$\vec{E} = m_l g = \rho_l V_l g = \rho_l \frac{4}{3}\pi R^3\, g$$

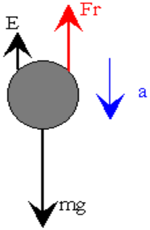
$$\rho_e V_e g - \rho_l V_l g - b\vec{v} = ma$$



$$\rho_e \frac{4}{3}\pi R^3\, g - \rho_l \frac{4}{3}\pi R^3\, g - b\vec{v} = m\frac{d\vec{v}}{dt}$$

$$\overrightarrow{F_S} = -b\vec{v}$$

http://www.sc.ehu.es/sbweb/fisica/dinamica/stokes/stokes.html



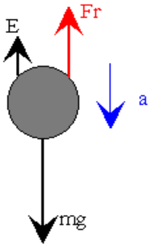
$$\rho_e \frac{4}{3}\pi R^3\, g - \rho_l \frac{4}{3}\pi R^3\, g - b\vec{v} = \rho_e \frac{4}{3}\pi R^3\, \frac{d\vec{v}}{dt}$$

Si despreciamos el Empuje

$$mg - b\vec{v} = m\frac{d\vec{v}}{dt}$$

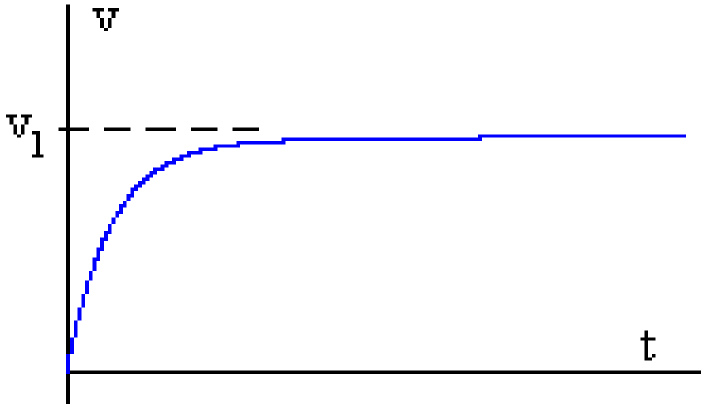
$$g - \frac{b}{m}\vec{v} = \frac{d\vec{v}}{dt} \quad \Rightarrow \quad dt = \frac{d\vec{v}}{g - \frac{b}{m}\vec{v}}$$

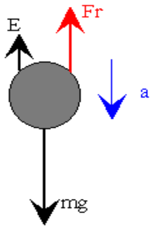
$$\frac{b}{m} = \beta \quad \Rightarrow \quad dt = \frac{d\vec{v}}{g - \beta\vec{v}}$$



$$\int_0^t dt = \int_0^{\mathrm{v}} \frac{d\vec{\mathrm{v}}}{g - \beta\vec{\mathrm{v}}}$$

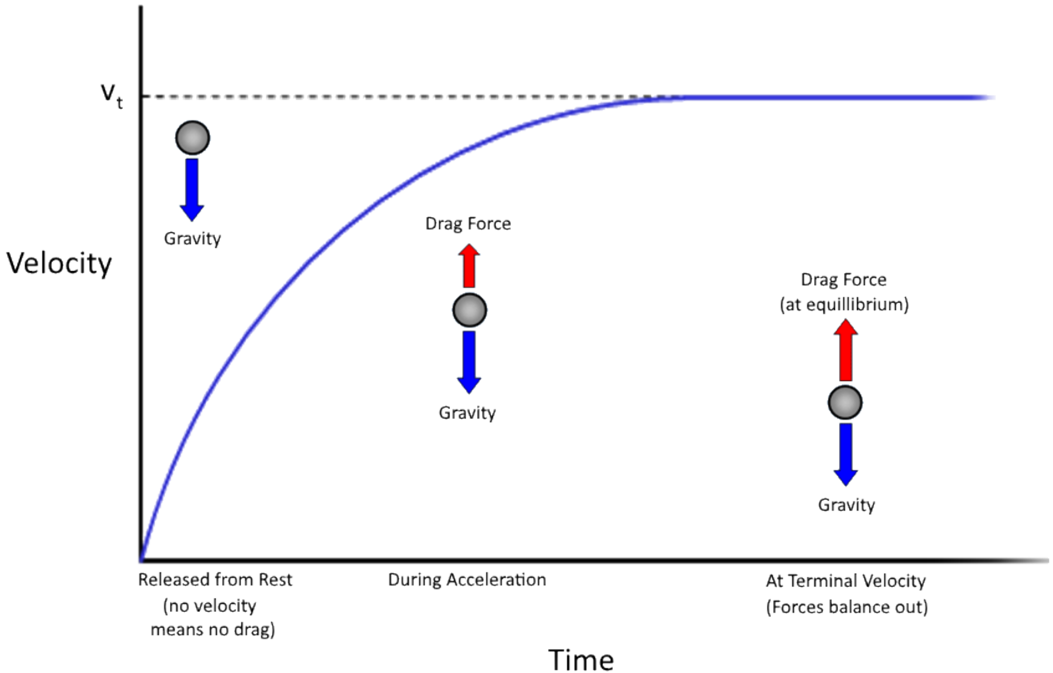
$$\mathrm{v} = \mathrm{v}_l \left[1 - e^{-\beta t}\right] \quad \text{donde} \quad \mathrm{v}_l = \frac{mg}{b}$$

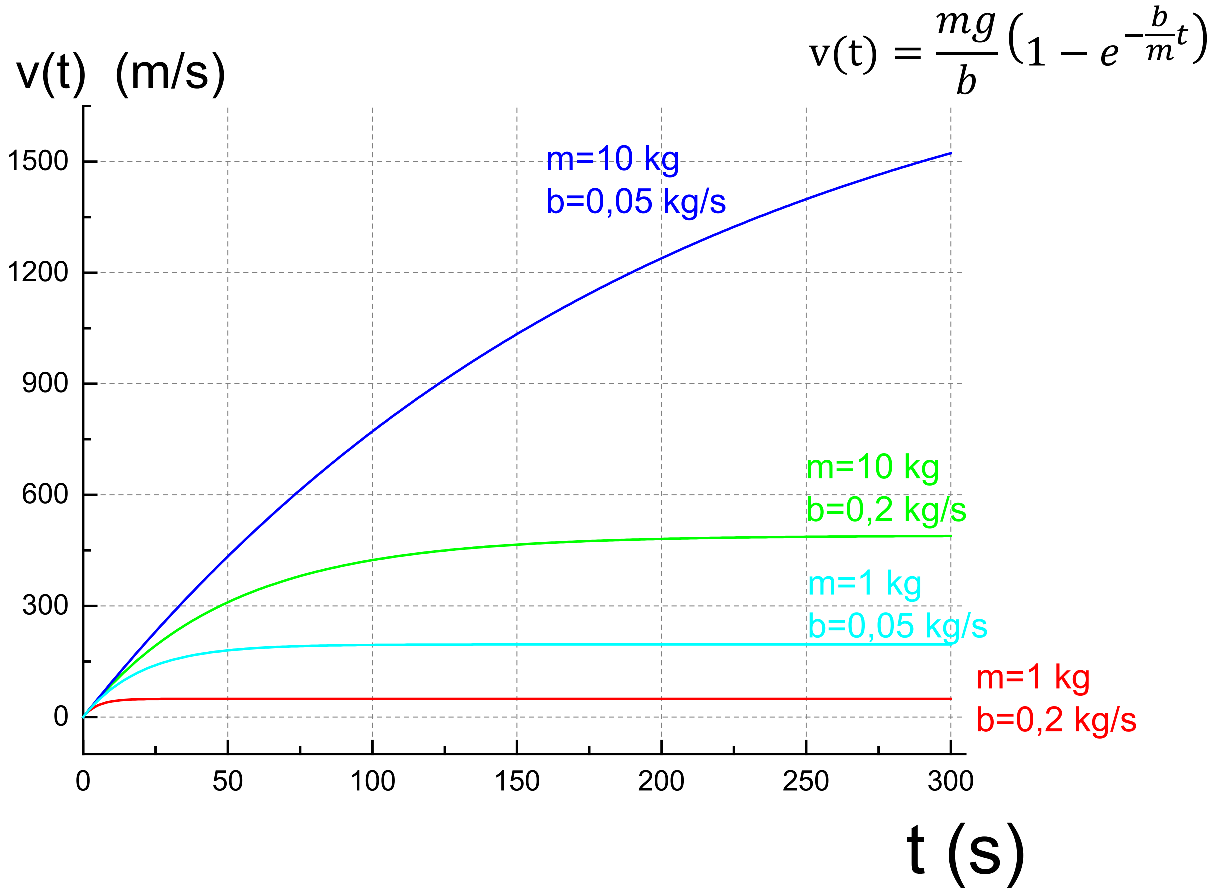




$$v(t) = v_l\left[1 - e^{-\beta t}\right] =$$

$$= \frac{mg}{b}\left[1 - e^{-\frac{b}{m}t}\right]$$





$v(t) = \frac{mg}{b}\left(1 - e^{-\frac{b}{m}t}\right)$
v(t) (m/s)
1500
1200
900
600
300
0
0
50
100
150
200
250
300
t (s)
m=10 kg
b=0,05 kg/s
m=10 kg
b=0,2 kg/s
m=1 kg
b=0,05 kg/s
m=1 kg
b=0,2 kg/s

Felix Baumgartner

supersonic stratosphere freefall

Fuerzas Especiales (Cont.)

On 14 October 2012, Baumgartner flew approximately **39 kilometres** (24 mi) into the stratosphere over New Mexico, United States, in a helium balloon before free falling in a pressure suit and then parachuting to Earth.

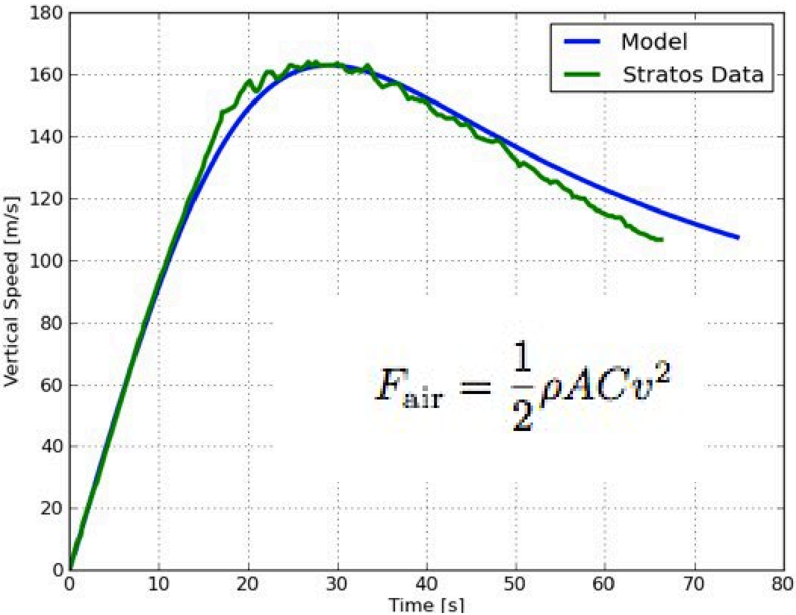
It took Baumgartner **about 90 minutes** to reach the target altitude and his free fall was estimated to have lasted **three minutes and 48 seconds** before his parachutes were deployed.

He sought to be the first free-falling human to **break** the sound barrier, which he **did** handily. His top speed was calculated at 833.9 miles per hour, or Mach 1.24. (The speed of sound is measured at 761.2 miles per hour at sea level.)

Fuerzas Especiales (Cont.)

I made some more assumptions:

- The drag coefficient (C) for the jumper was constant.
- I calculated the product of the drag coefficient (C) and object surface area (A) based on the terminal speed of a skydiver at about 120 mph. It was just a guess.
- The density of air (ρ) decreased with altitude based on this model described in Wikipedia.
- I originally used the universal ($1/r^2$) model for the gravitational force, but I am pretty sure you could just use a constant mass times little g where g is about 9.8 N/kg.



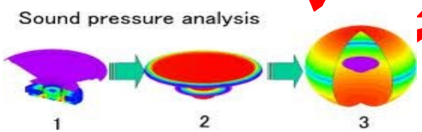
$$F_{\mathrm{air}} = \frac{1}{2}\rho A C v^2$$

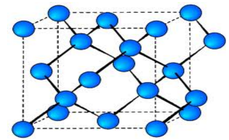
https://www.wired.com/2012/07/analysis-of-a-red-bull-stratos-practice-jump/

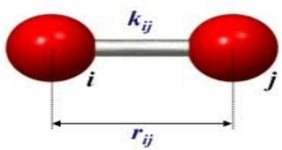
SPECIAL FORCES

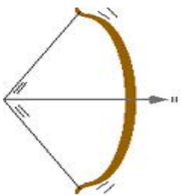
Fuerzas Especiales (Cont.)

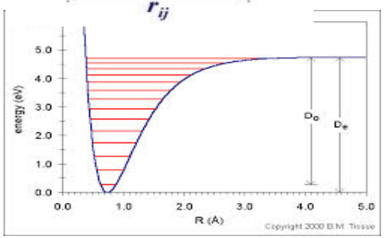
✓ Fuerza elástica.

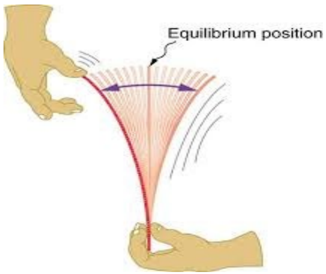


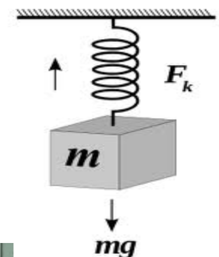






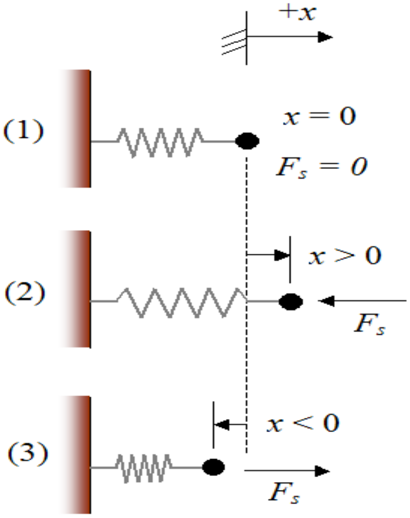




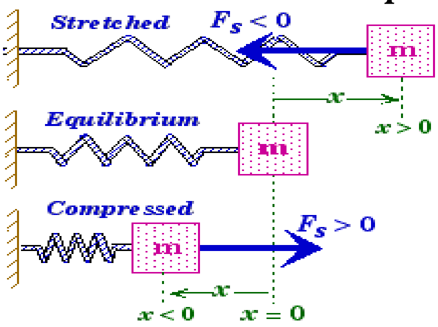


La fuerza restauradora es

$$F = -k\Delta x \qquad k > 0$$

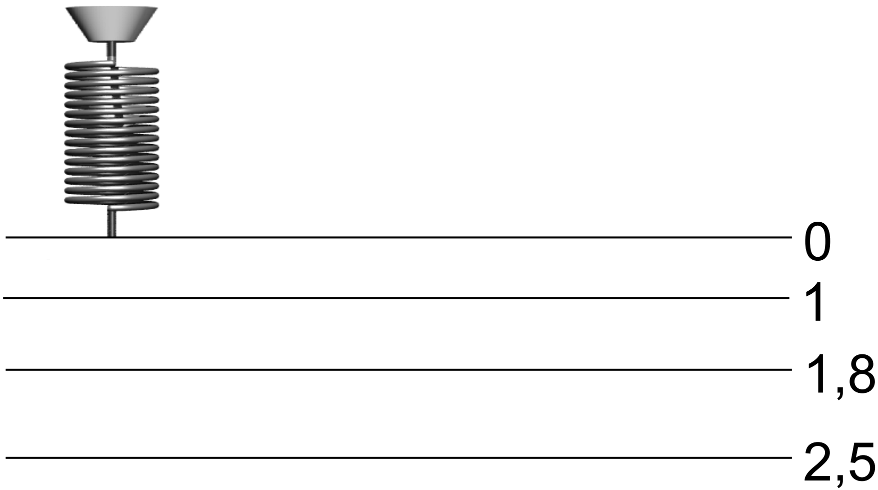


$$F = -k(x - x_0)$$

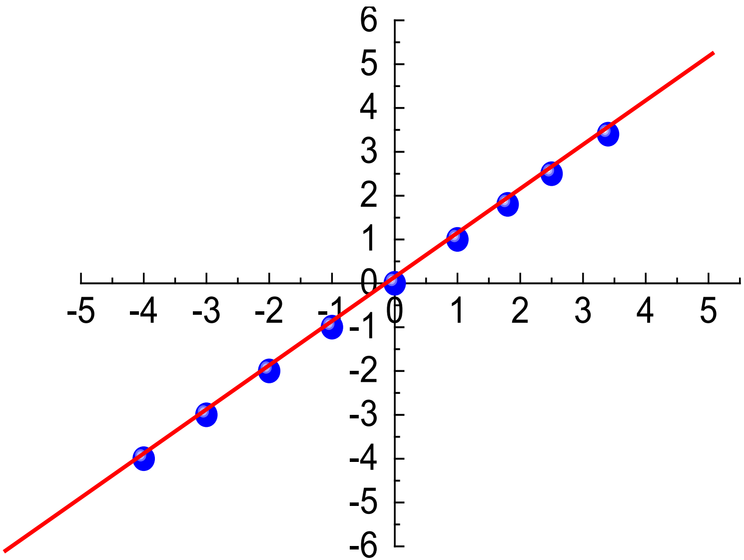


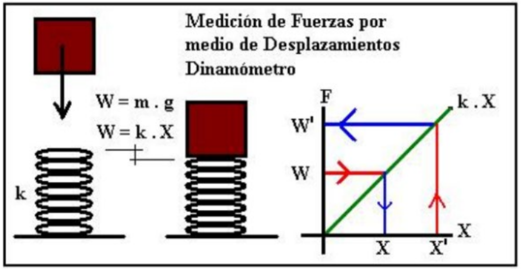
$$F_s = -k\,x$$

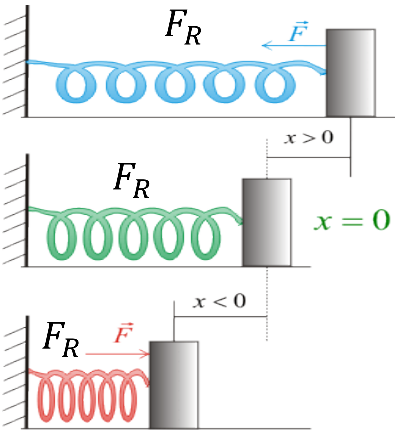
Qué significa $F = -k\Delta x$?



Qué significa $F = -k\Delta x$?







$$\sum_{i=1}^{N} F_{ext} = ma \begin{cases} \sum_{i=1}^{N} F_x = F_R = ma \\ \sum_{i=1}^{N} F_y = N - mg = 0 \end{cases}$$

$$-kx = ma_x = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

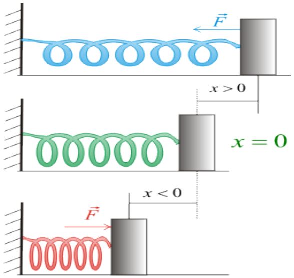
Esta es la ecuación diferencial del movimiento del bloque

Si llamo $\frac{k}{m} = \omega^2 \ldots$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Ecuación diferencial del oscilador armónico

La solución de esta ecuación diferencial ordinaria deberá ser una función **cuya segunda derivada sea la misma función con el signo invertido**.
Las únicas funciones (no complejas) que satisfacen esta condición son sen(x) y cos(x).
Pero estas dos opciones están relacionadas ya que $sen(x + \pi/2) = \cos(x)$
Por eso, escogemos arbitrariamente cos(x), sabiendo que podremos cambiar a sen(x) sumando una constante.
La solución se escribe:



$$x(t) = A\cos(\omega t + \delta)$$

Comprobemos que es una solución de la ecuación diferencial:

$$\frac{d}{dt} A\cos(\omega t + \delta\ ) = -\omega A\, sen(\omega t + \delta\ )$$

$$\frac{d^2}{dt^2} A\cos(\omega t + \delta\ ) = \frac{d}{dt} - \omega A\, sen(\omega t + \delta\ ) =$$

$$= -\omega^2 A\, cos(\omega t + \delta\ )$$

Entonces

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$-\omega^2 A\, cos(\omega t + \delta\ ) + \omega^2\{A\cos(\omega t + \delta\ )\} = 0$$

Se cumple para todo valor de t.

Si temenos la solución de

$$x(t) = A\cos(\omega t + \delta\ )$$

Entonces Tambien temenos la velocidad y la aceleración en todo instante

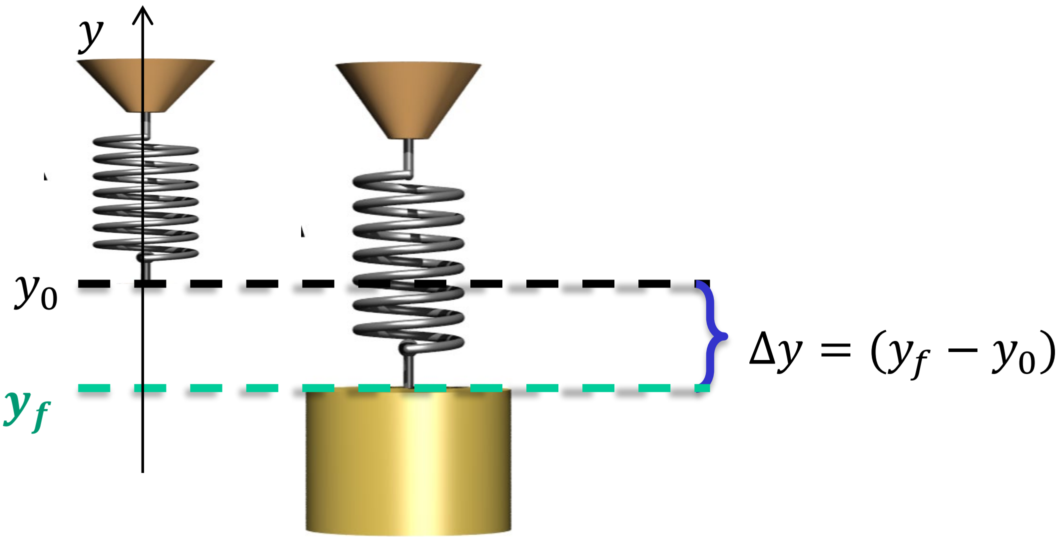
$$v(t) = \frac{d}{dt}A\cos(\omega t + \delta\ ) = -\omega A\, sen(\omega t + \delta\ )$$

$$a(t) = \frac{d^2}{dt^2}A\cos(\omega t + \delta\ ) = \frac{d}{dt} - \omega A\, sen(\omega t + \delta\ ) =$$

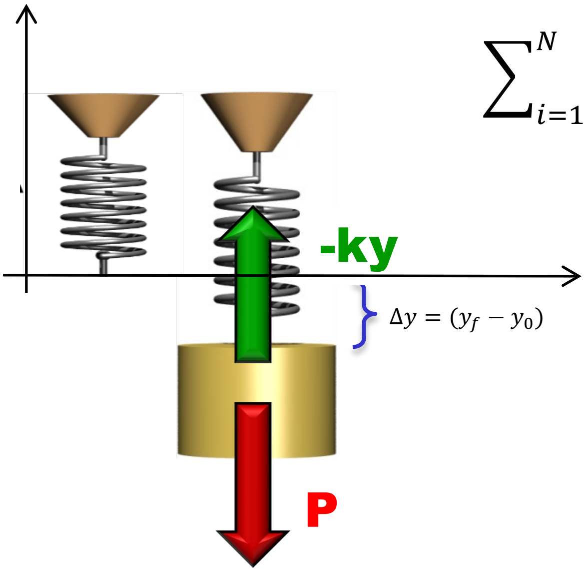
$$= -\omega^2 A\, cos(\omega t + \delta\ )$$

Se cumple para todo valor de t.

EJEMPLO I



EJEMPLO I



$$\sum_{i=1}^{N} F_{ext} = ma \begin{cases} \sum_{i=1}^{N} F_x = 0 + 0 + 0 = 0 \\ \sum_{i=1}^{N} F_y = -ky - mg = ma_y \end{cases}$$

$$Cuando\ \sum_{i=1}^{N} F_y = 0$$

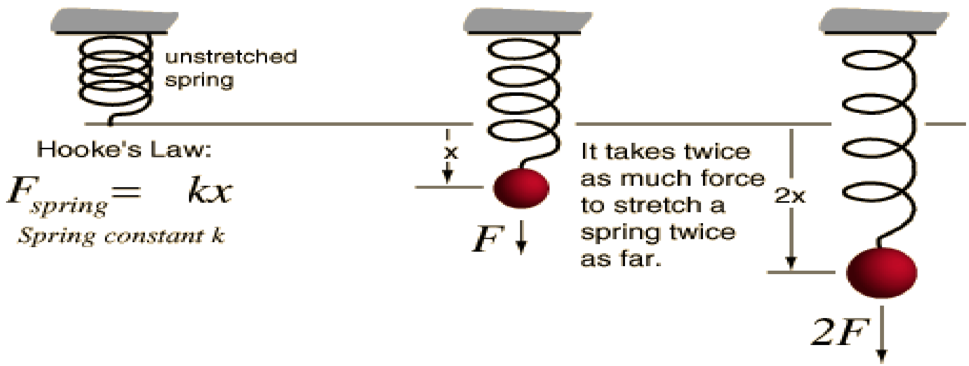
$$-ky_f - mg = 0$$

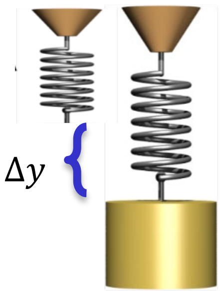
$$\boldsymbol{y_f = -\frac{mg}{k}} \qquad \boldsymbol{y_f < 0}$$

$Si\ M = 2m, cuando\ \sum_{i=1}^{N} F_y = 0$

$-ky_{fM} - Mg = 0$

$$\boldsymbol{y_{fM} = -\frac{2mg}{k}} \qquad \boldsymbol{y_{fM} = 2y_{fm}}$$





$$\sum_{i=1}^{N} F_{ext} = ma \begin{cases} \sum_{i=1}^{N} F_x = 0 \\ \sum_{i=1}^{N} F_y = -ky - mg = ma_y \end{cases}$$

$$-ky - mg = m\frac{d^2y}{dt^2}$$

$$\frac{d^2y}{dt^2} + \frac{k}{m}y + g = 0$$

$$\frac{k}{m} = \omega^2$$

$$\frac{d^2y}{dt^2} + \omega^2 y + g = 0$$

$$y' = y + \Delta L$$

$$\frac{k}{m} = \omega^2$$

$$\sum_{i=1}^{N} F_{ext} = ma \begin{cases} \sum_{i=1}^{N} F_x = 0 \\ \sum_{i=1}^{N} F_y = -ky' - mg = ma_y \end{cases}$$

$$-ky' = -k(y + \Delta L) - mg = m\frac{d^2y'}{dt^2}$$

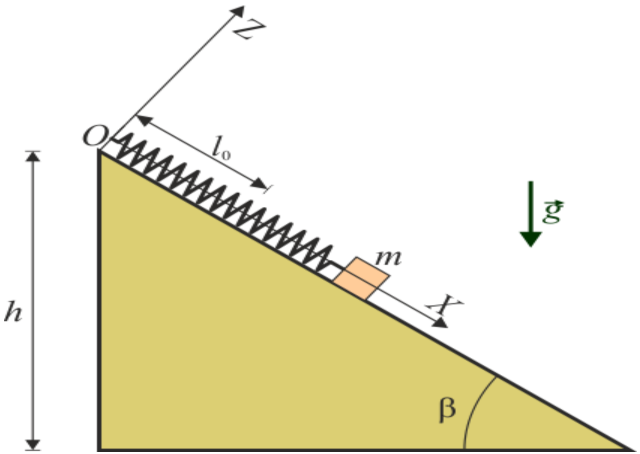
$$= -ky - k\Delta L - mg = m\frac{d^2y'}{dt^2}$$

de la condicion de equilibrio $-k\Delta L - mg = 0$

Entonces $\dfrac{d^2y'}{dt^2} + \dfrac{k}{m}y' = 0$

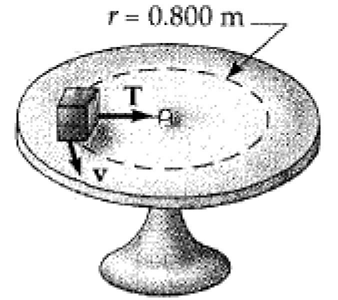
$$\frac{d^2y'}{dt^2} + \omega^2 y' = 0$$

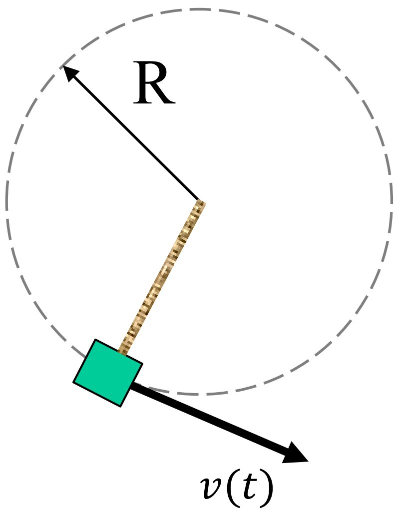
EJEMPLO II



Utilizar la estrategia del Problema I y elegir el sistema de coordenadas en la posicion de equilibrio del bloque.

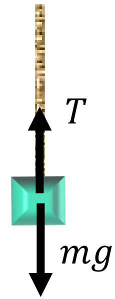
Se sabe que una cuerda puede soportar una carga máxima colgada de 25 kg antes de romperse. A dicha cuerda se la ata una masa de 3 kg y se la hace girar en una mesa horizontal sin rozamiento en un circulo de 0,8 metros de radio como muestra la figura. Cual es la máxima velocidad a la que puede girar dicha masa antes de romper la cuerda?

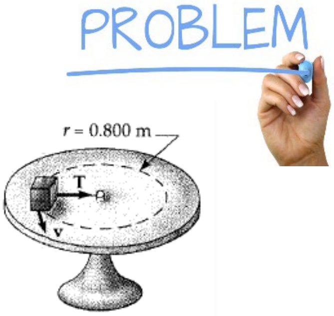




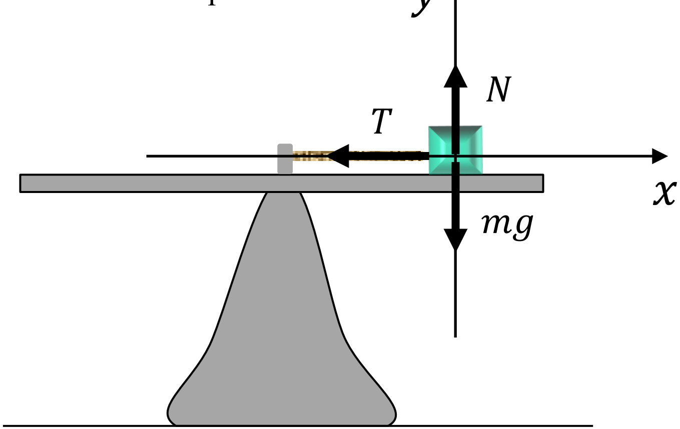
Carga máxima

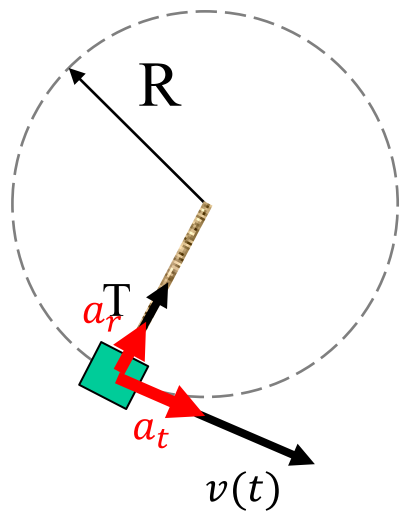
$$T = mg = 25\ kg\ \times 10 \frac{m}{s^2} = 250\ N$$





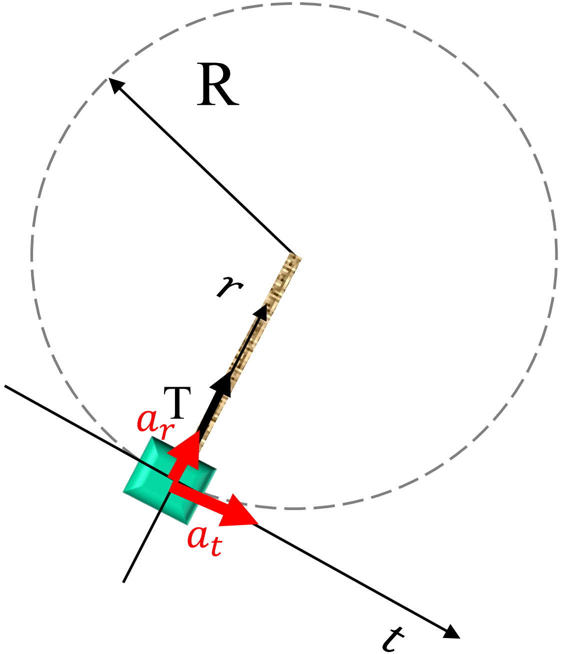
Sistema: Bloque

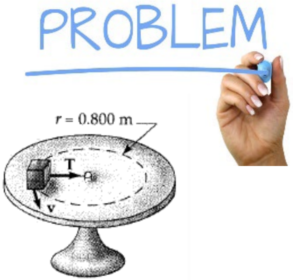




Sistema: Bloque

$$\sum_{i=1}^{N} F_{ext} = ma$$





$$\begin{cases} \sum_{i=1}^{N} F_r = ma_r \\ \sum_{i=1}^{N} F_t = 0 \end{cases}$$

$$\sum_{i=1}^{N} F_r = T = ma_r$$

$$T = ma_r = m\frac{v^2}{R}$$

$$250\ N = 3\ kg\,\frac{v^2}{0{,}8\ m}$$

$$v = \sqrt{\frac{0{,}8\ m \times 250\ N}{3\ kg}} = 66{,}66\,\frac{m}{s}$$

The End