CINEMÁTICA

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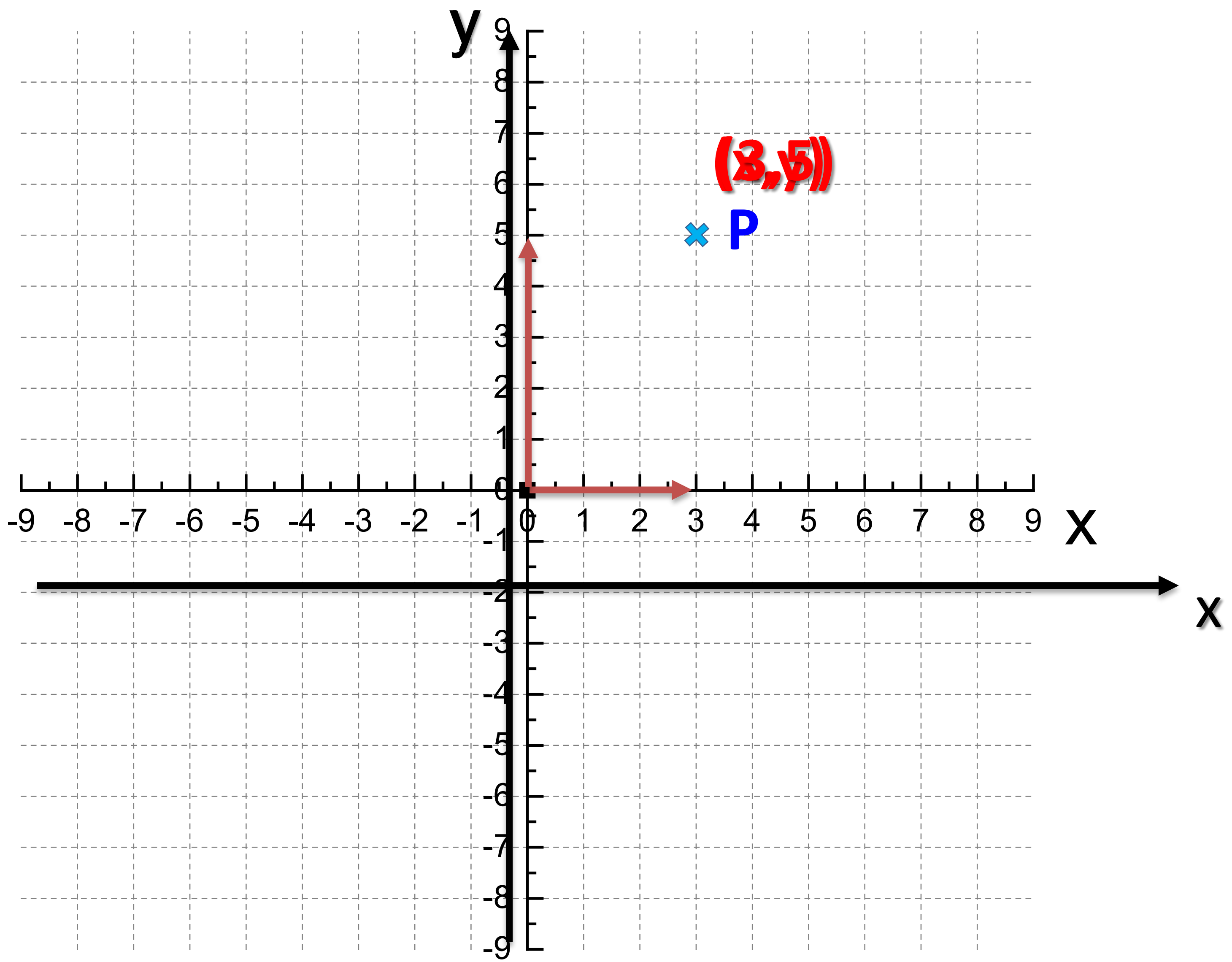
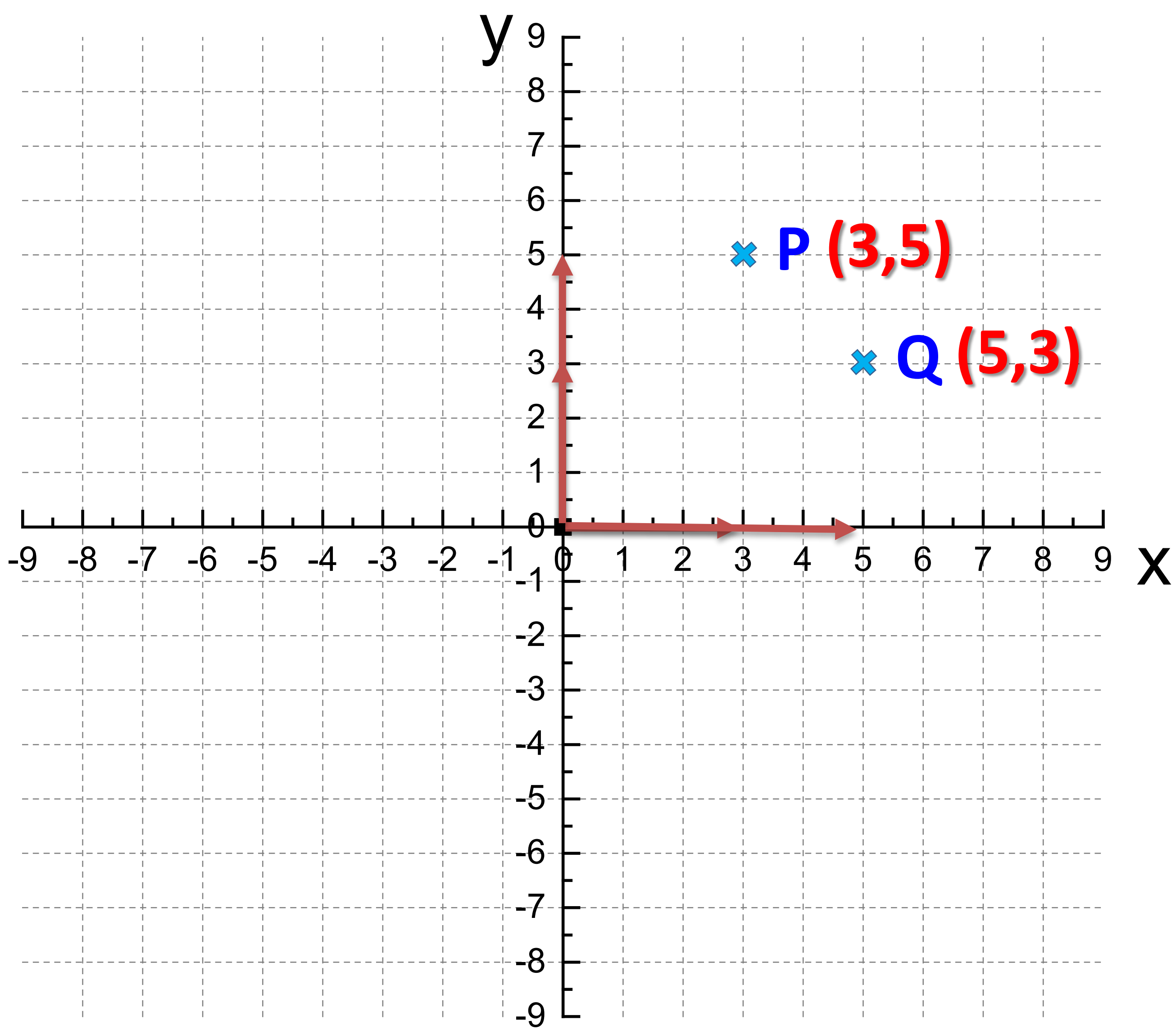
Qué vamos a ver

- Posición, velocidad, aceleración.
  - Modelo. Magnitud. Problemas. Soluciones.
  - Coordenadas cartesianas vs. polares
- Movimiento uniformemente acelerado.
  - Ecuaciones que describen el MUA.
  - Algunos casos en los que la aceleración es variable.
- Movimiento en 2 y 3 dimensiones
  - Caída libre. Tiro oblicuo.
  - Casos de aplicación.
  - Problemas numéricos

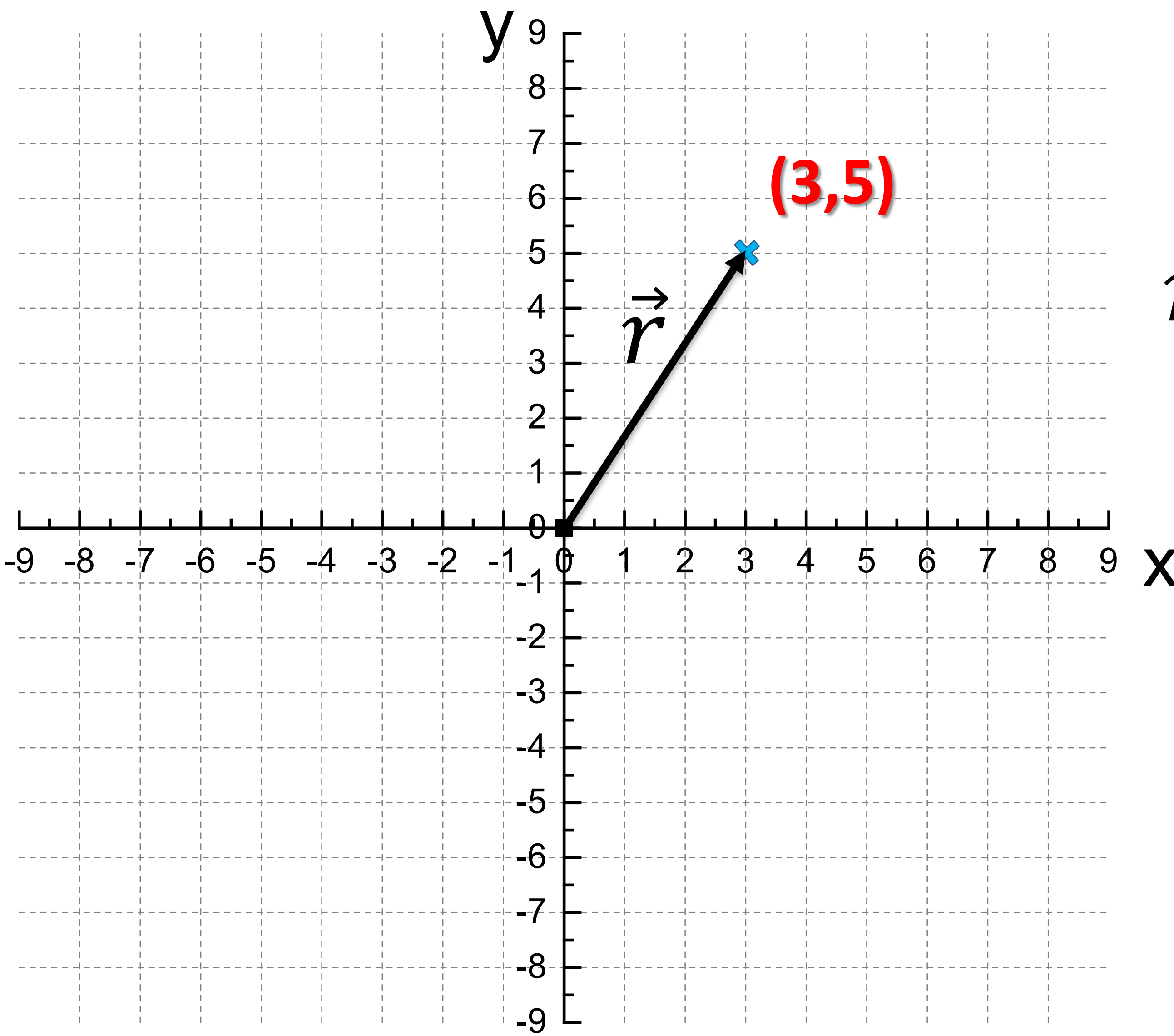
Vector posición y vector desplazamiento



Vector posición y vector desplazamiento



Vector posición y vector desplazamiento

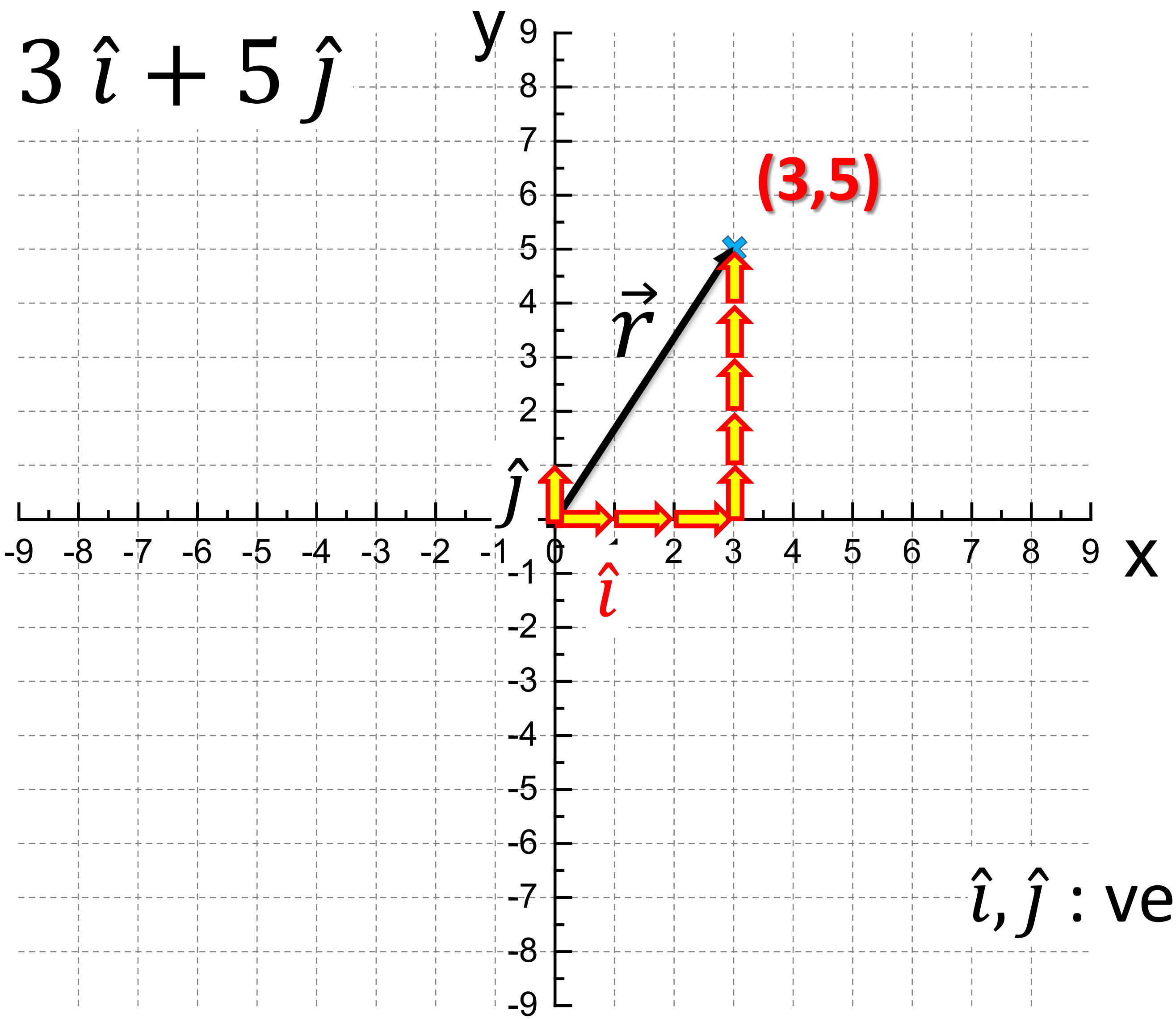


P=(3,5)

$\vec{r}=(3,5)$

Vector posición y vector desplazamiento

$$\vec{r} = 3\,\hat{\imath} + 5\,\hat{\jmath}$$



$\hat{\imath}, \hat{\jmath}$: versores

VECTORES

Vectores

- **Qué es un vector. Cuándo es necesario.**
- **Composición vectorial**
- **Suma, resta.**
  Productos (escalar y vectorial).

$$\vec{F} = F_x\,\hat{i} + F_y\,\hat{j} + F_z\,\hat{k}$$

$$\vec{r} = r_x\,\hat{i} + r_y\,\hat{j} + r_z\,\hat{k}$$

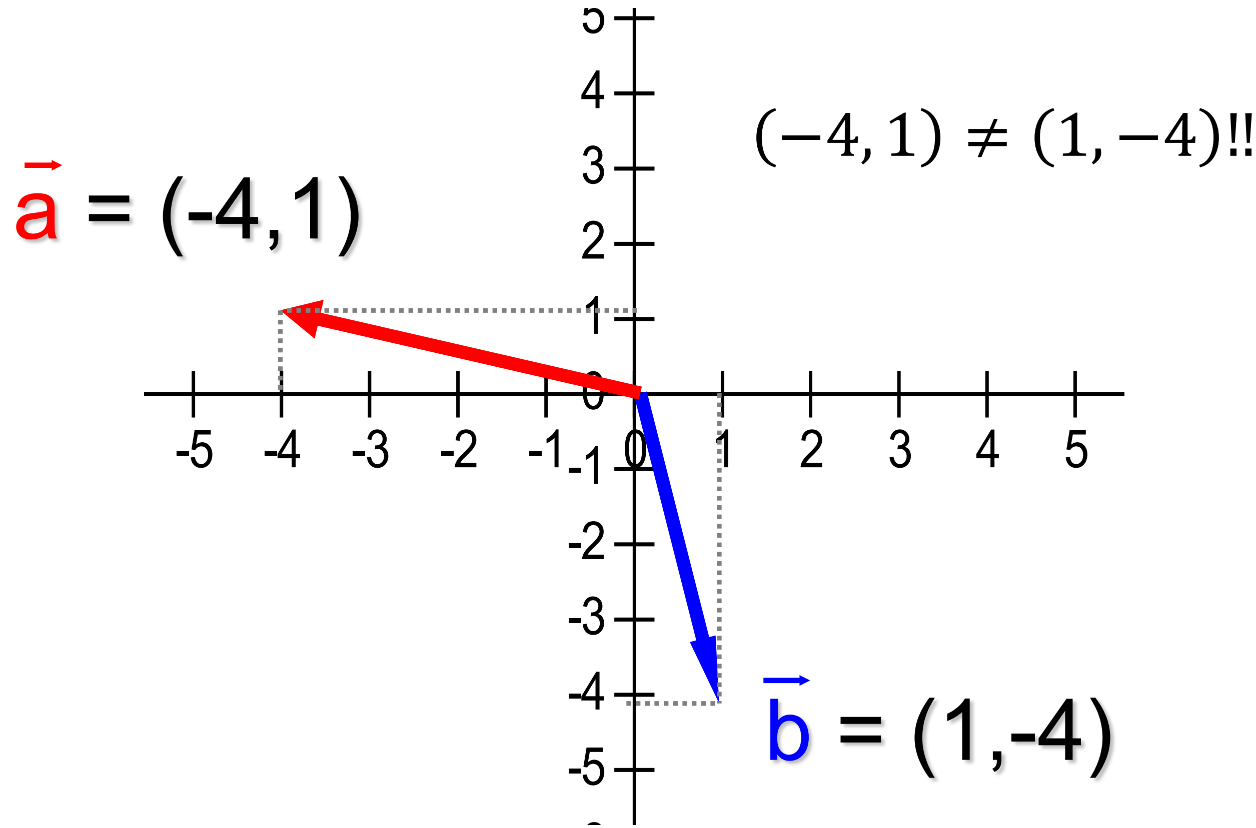
$$\vec{F} = (F_x\,, F_y\,, F_z\,)$$

$$\vec{r} = (r_x\,, r_y\,, r_z\,)$$

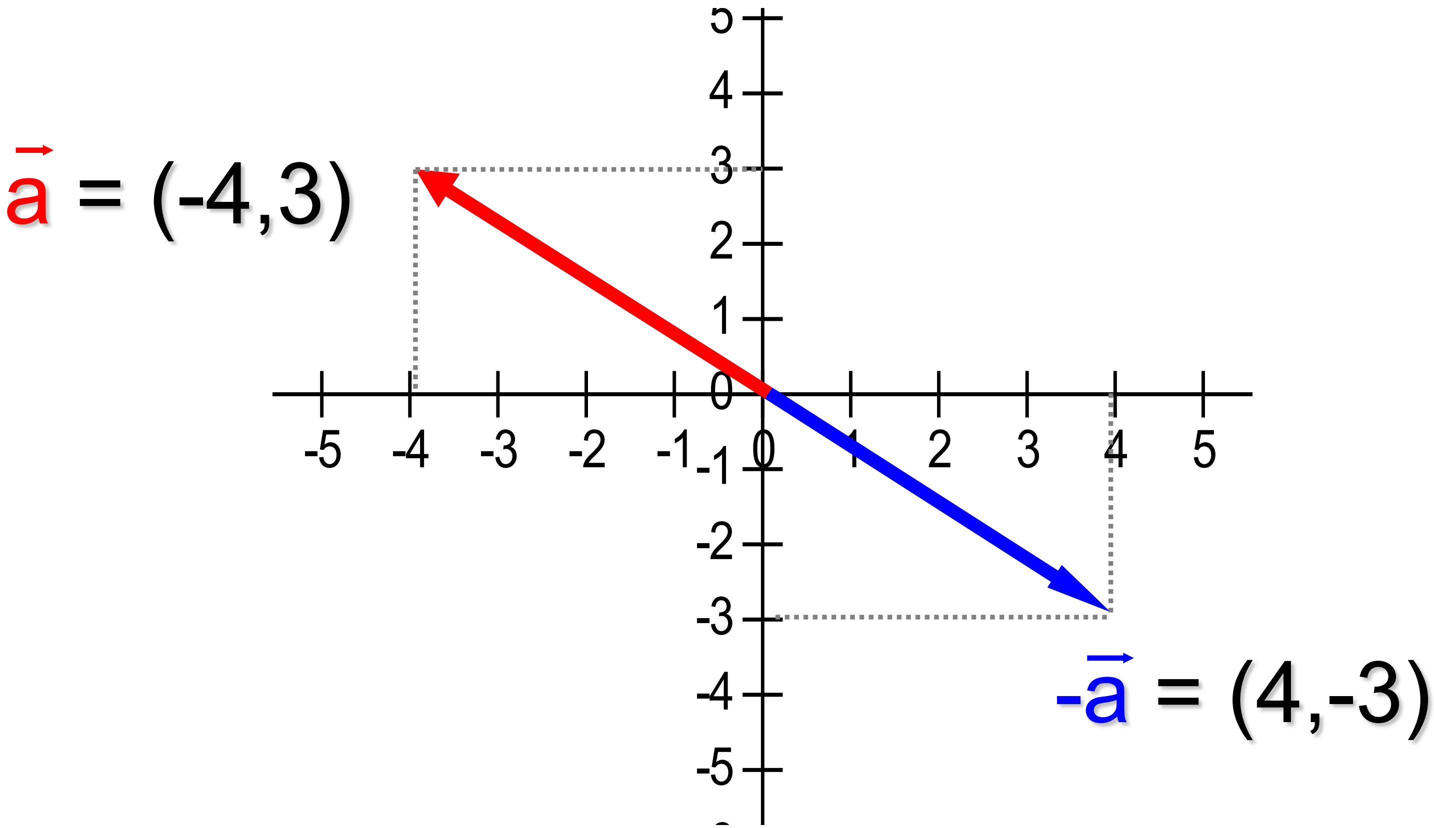
$$\vec{F} = \left[\left|F\right|, \theta\,\right]$$

$$\vec{r} = \left[\left|r\right|, \alpha\,\right]$$

$$(x, y) \neq (y, x)!!$$

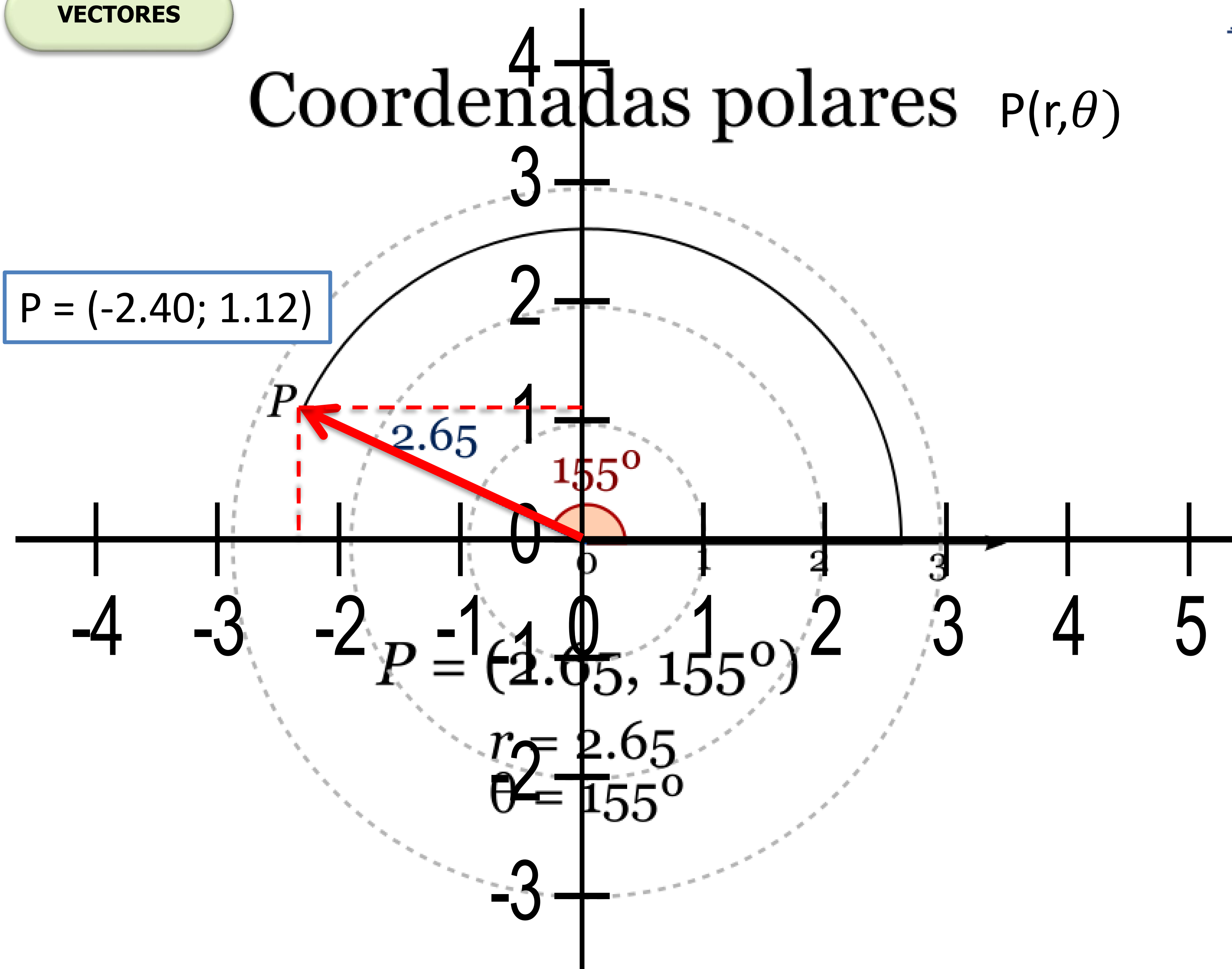


VECTORES



Coordenadas polares $P(r,\theta)$

P = (-2.40; 1.12)



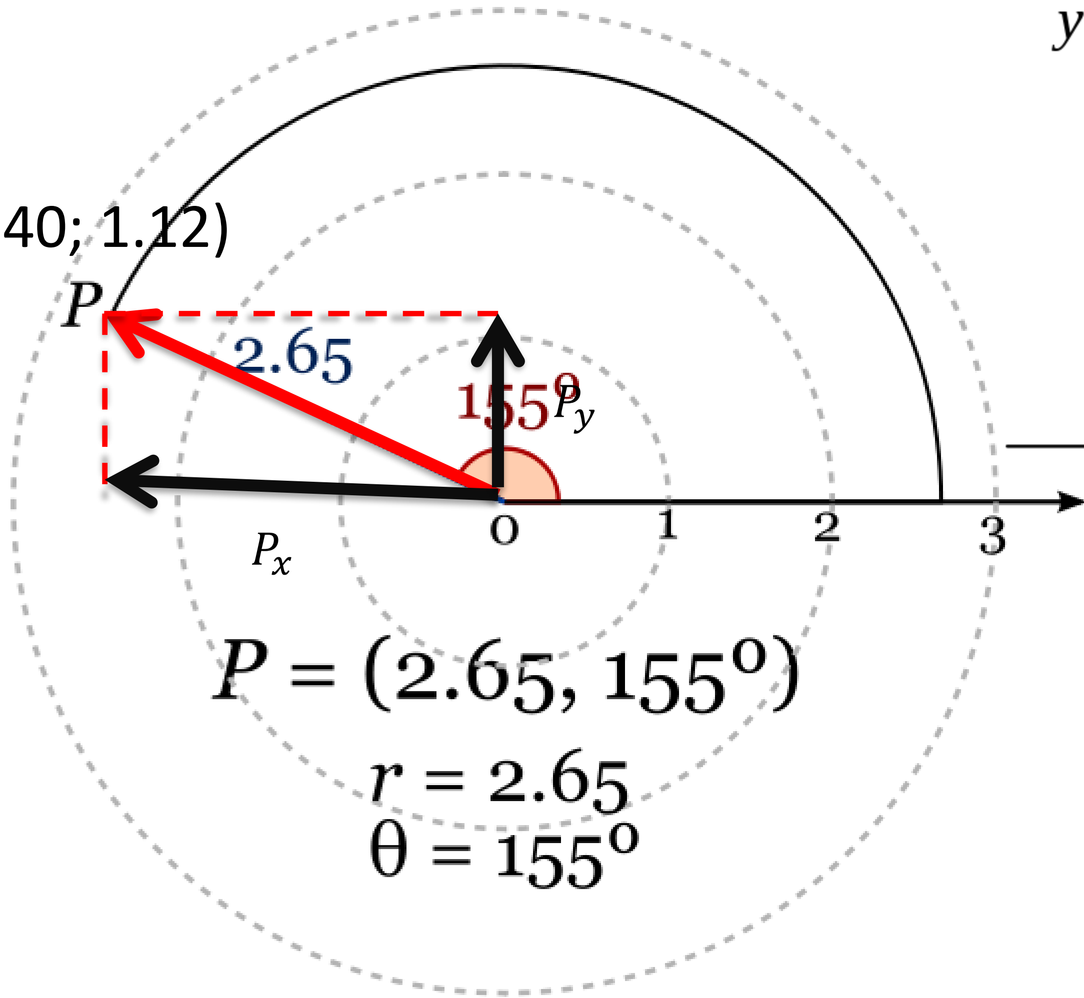
$$P = (2.65, 155^o)$$

$$r = 2.65$$

$$\theta = 155^o$$

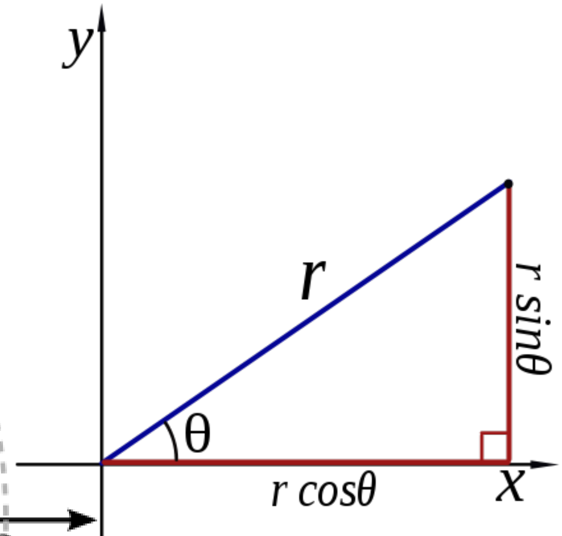
Coordenadas polares $P(r,\theta)$

P = (-2.40; 1.12)

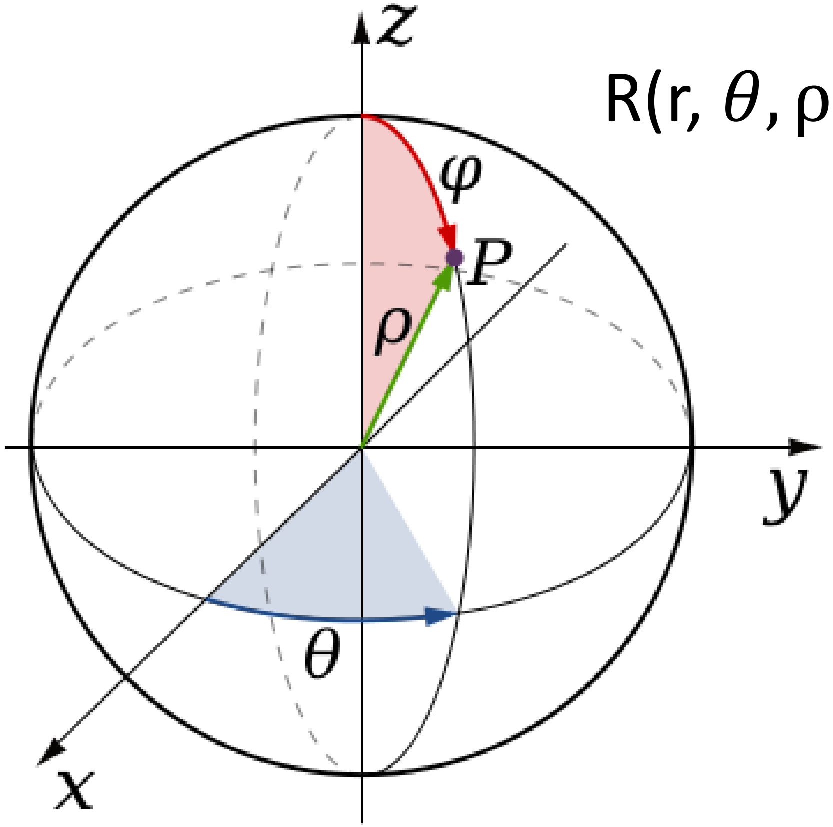


$$P = (2.65, 155^o)$$

$$r = 2.65$$
$$\theta = 155^o$$



Coordenadas Esféricas



R(r, θ, ρ)

Producto Escalar de dos Vectores

- Definición: $\mathbf{W} = \vec{\mathbf{F}} \cdot \vec{\mathbf{r}}$

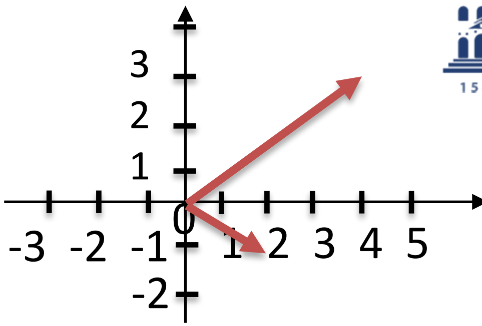
$\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath}$ $\vec{r} = r_x\hat{\imath} + r_y\hat{\jmath}$	$\vec{F} = (F_x, F_y)$ $\vec{r} = (r_x, r_y)$	$\vec{F} = [\lvert F \rvert, \theta]$ $\vec{r} = [\lvert r \rvert, \alpha]$

$$\vec{F} \cdot \vec{r} = F_x r_x + F_y r_y$$

o bien

$$\vec{F} \cdot \vec{r} = |F| \, . |r| \, . \cos\theta$$

Producto Escalar de dos Vectores



- Definición: $\mathbf{W} = \vec{\mathbf{F}} \cdot \vec{\mathbf{r}}$

$\vec{F} = 3\hat{\imath} + 4\hat{\jmath}$	$\vec{F} = (3{,}4)$	$\vec{F} = [\lvert 5 \rvert, 53{,}1º]$
$\vec{r} = -1\hat{\imath} + 2\hat{\jmath}$	$\vec{r} = (-1{,}2)$	$\vec{r} = [\lvert\sqrt{5}\rvert, 116{,}6]$

$$\vec{F} \cdot \vec{r} = 3 \times (-1) + 4 \times 2 = 5$$

o bien

$$\vec{F} \cdot \vec{r} = |5| \cdot |2{,}23|\ 0{,}448 = 5$$

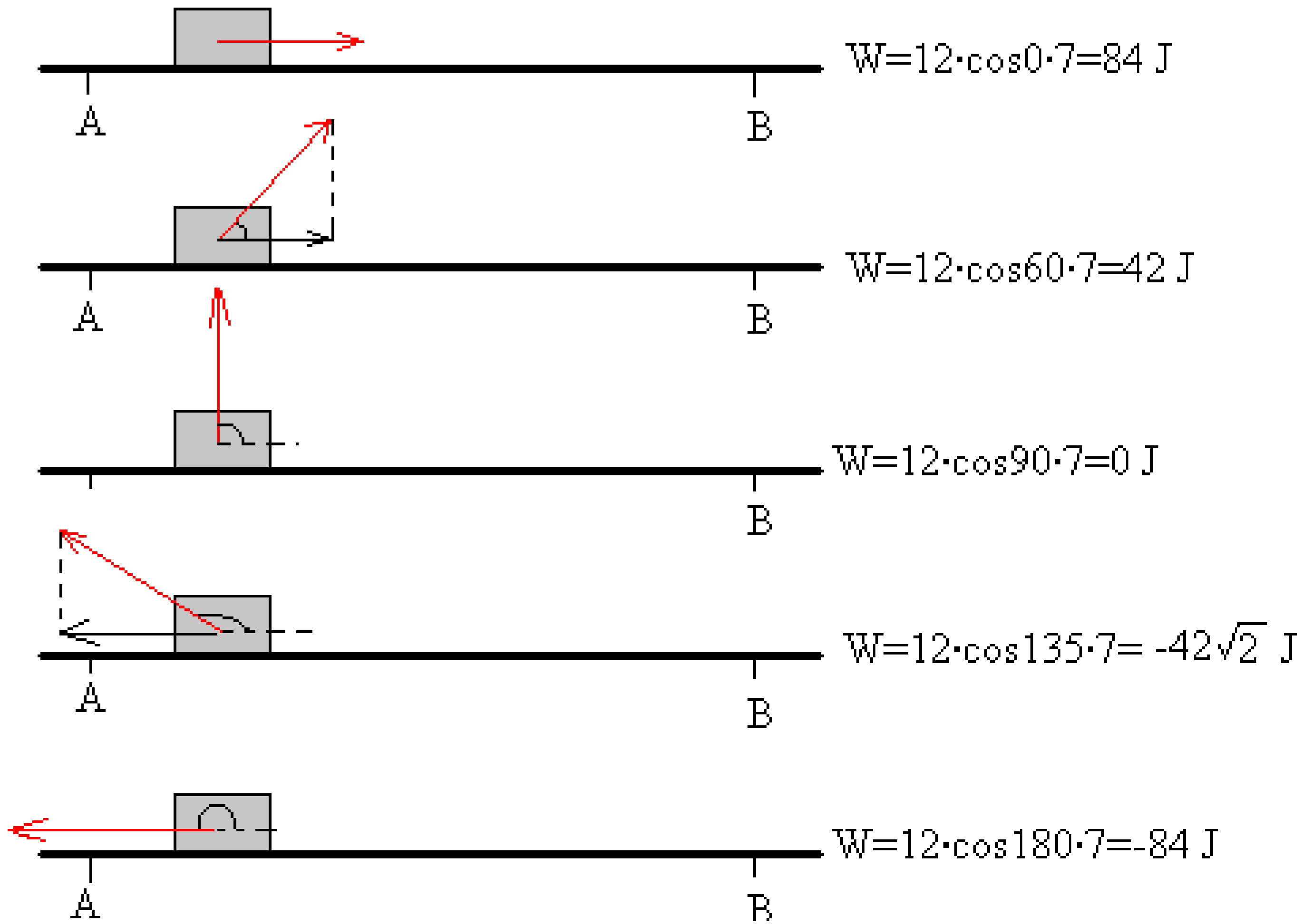
Trabajo

Entonces, si $\vec{F}$ = cte,

$$W = \vec{F} \cdot \vec{r} \Longrightarrow W = F \cdot r \cos\theta$$

F = 12 N

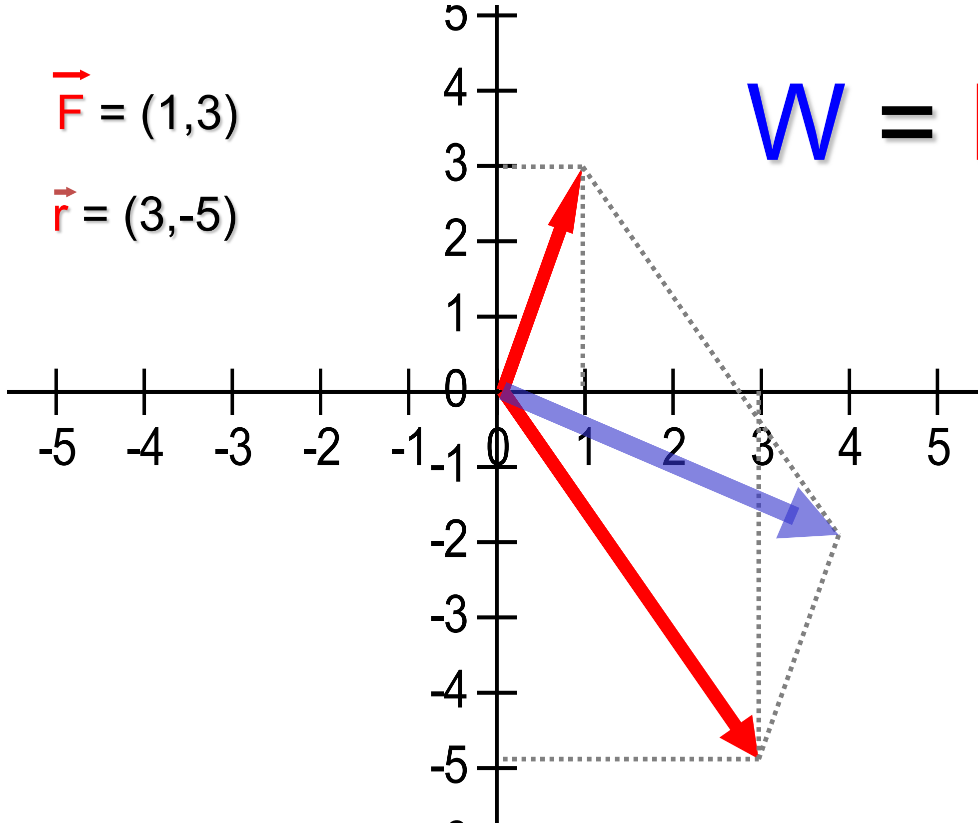
r = 7 m



$\vec{F} = (1,3)$

$\vec{r} = (3,-5)$

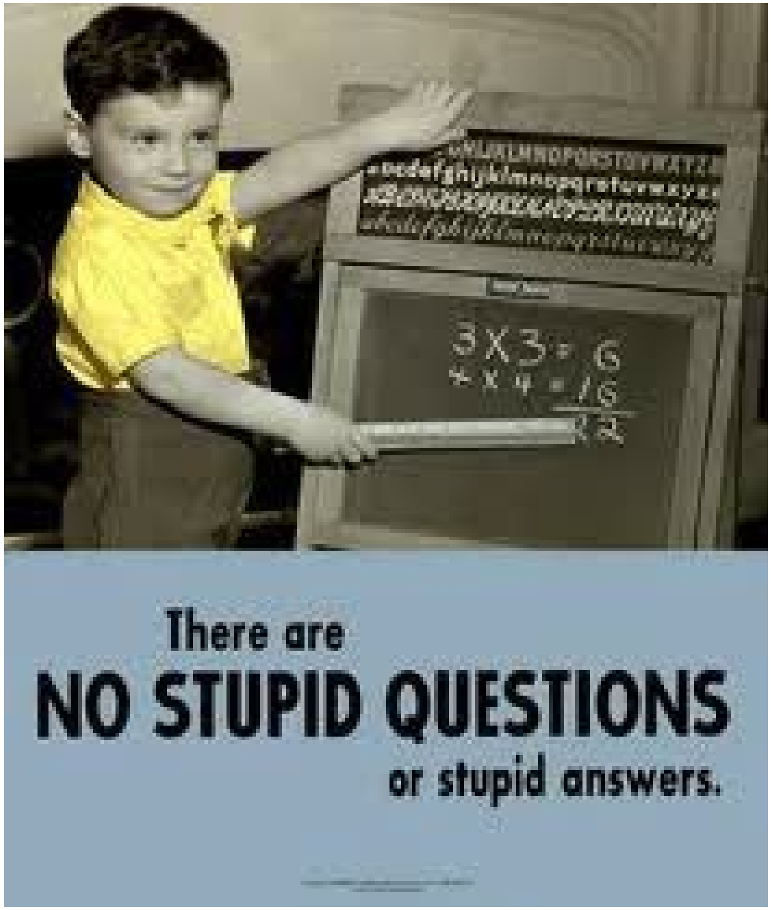
$$W = \vec{F}.\vec{r}$$



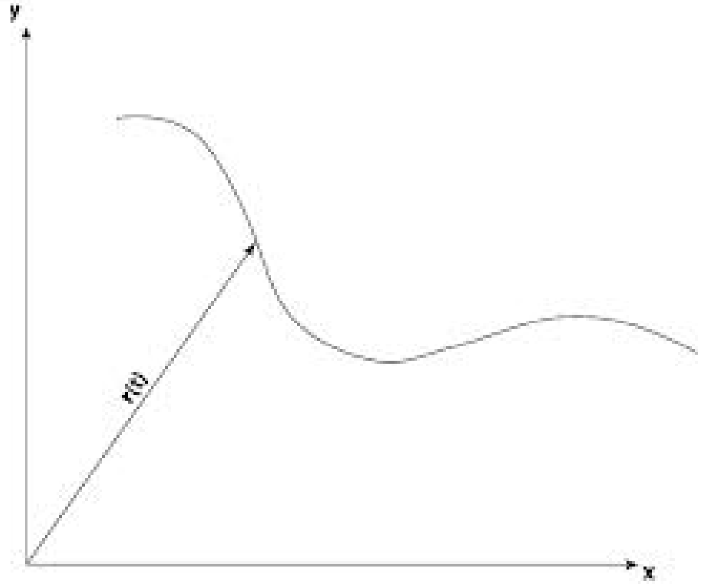
$$\vec{F}.\vec{r} = F_x.r_x + F_y.r_y \quad = 3.1+3.(-5)=3-15=-12$$

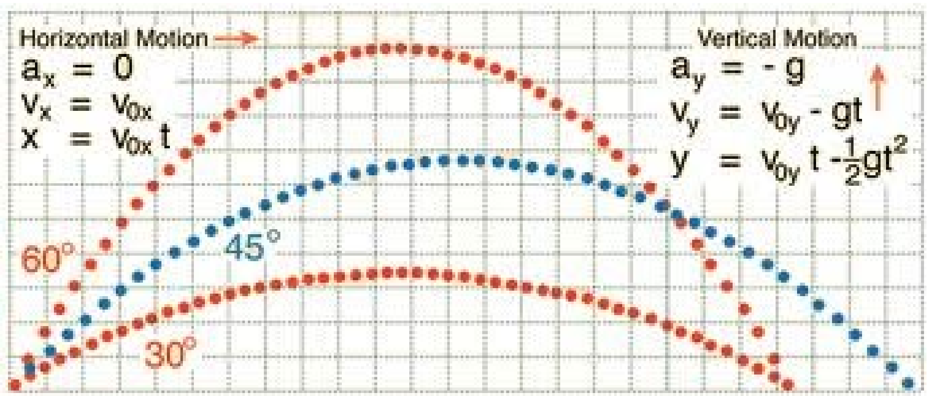
$$\vec{F}.\vec{r} = |F|\,.|r|\,.\,\cos\theta \quad = \sqrt{10}.\sqrt{36}.\cos(130.6) = -12$$

Espacio 'silly question'



Vector posición y vector desplazamiento





- Posición instantánea

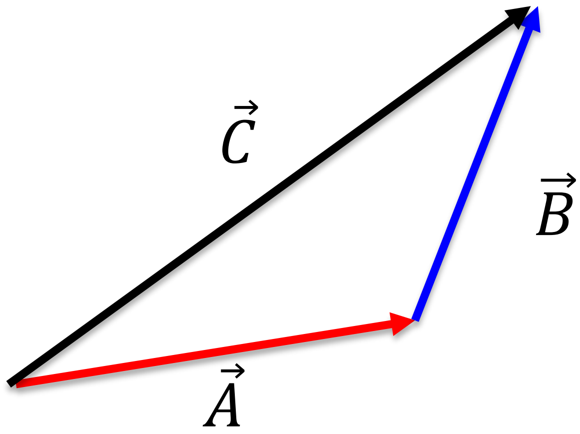
$$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$$

- Ecuación de la trayectoria

$$x = x(t), \qquad y = y(t), \qquad z = z(t)$$

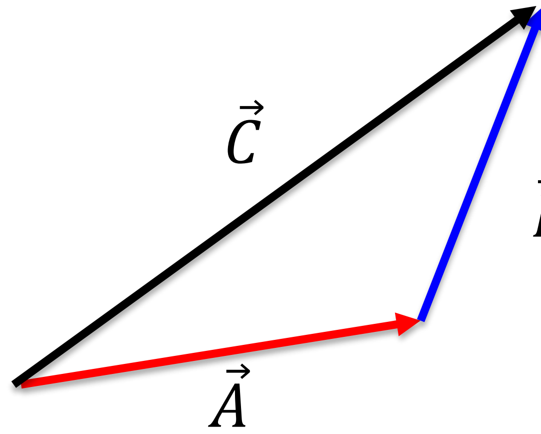
- Suma y Resta de vectores

$$\vec{C} = \vec{A} + \vec{B}$$



- Suma y Resta de vectores

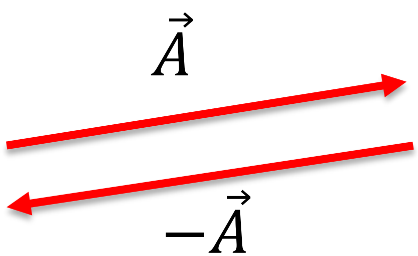
$$\vec{C} = \vec{A} + \vec{B}$$



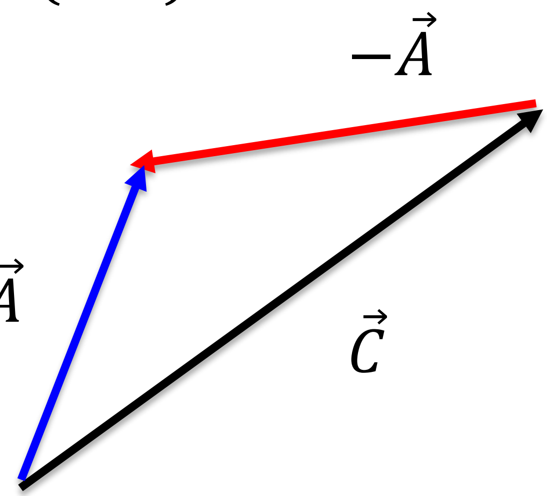
$$\vec{B} = \vec{C} - \vec{A}$$

Para restar sume el negativo

$$\vec{B} = \vec{C} + (-\vec{A})$$

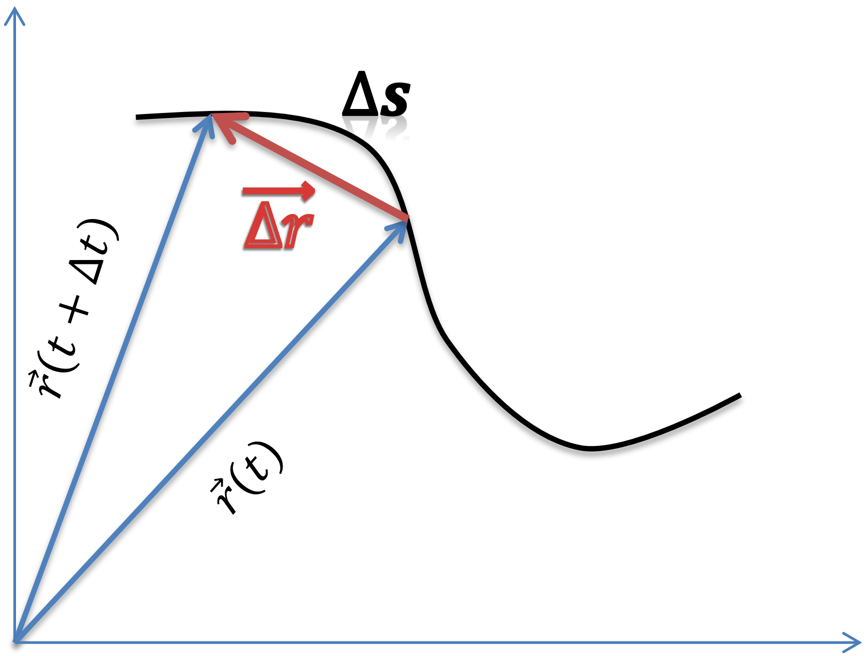


$$\vec{B} = \vec{C} - \vec{A}$$



- Vector desplazamiento

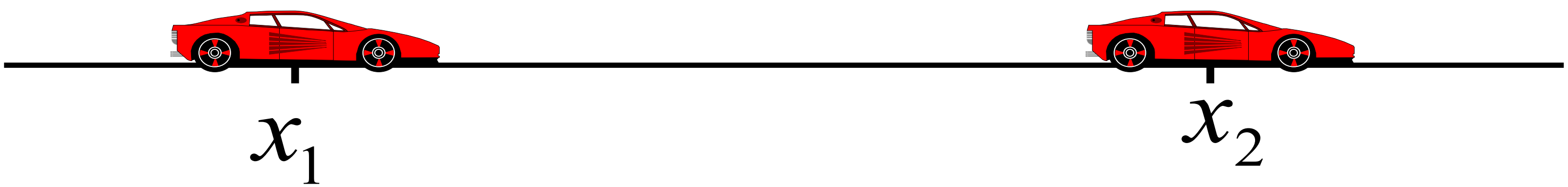
$$\overrightarrow{\Delta r}(t) = \vec{r}(t + \Delta t) - \vec{r}(t)$$



Movimento em una Dimensión (Cinemática)

- Trabajamos con ***Particulas***

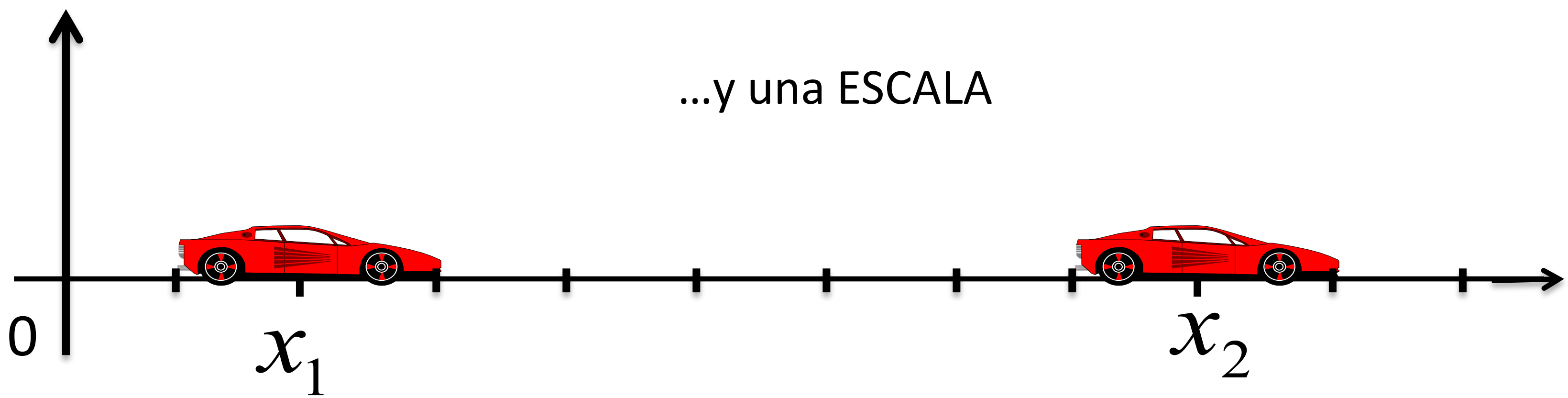
Posición



Cuál es el valor de x_1 (y de x_2) ?? → Precisamos un *Sistema de Referencia*

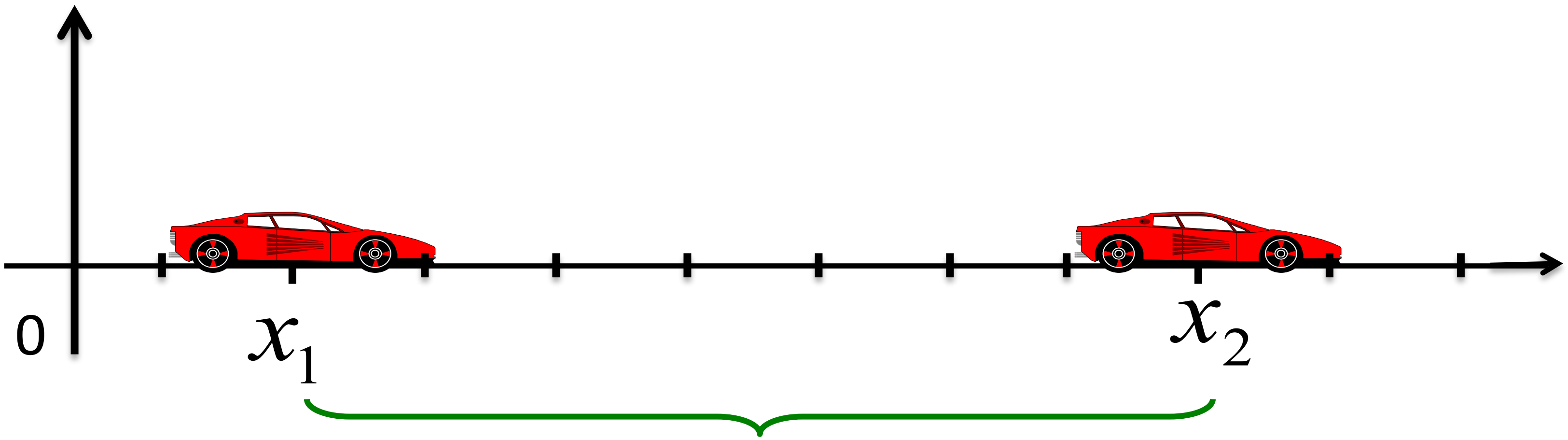
Un ORIGEN …

…y una ESCALA



Movimento em una Dimensión (Cinemática)

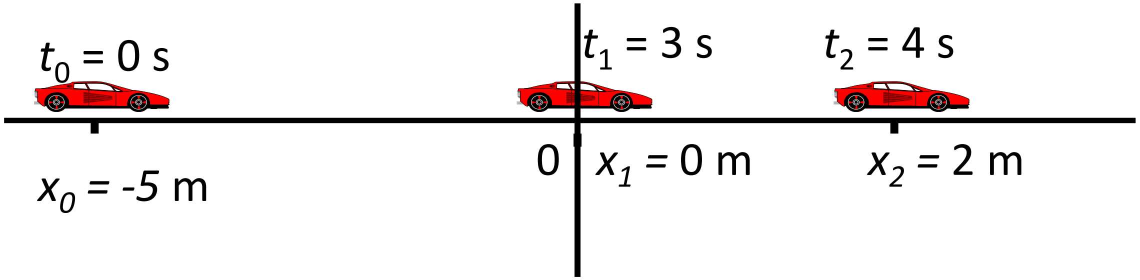
Desplazamiento:



$$\Delta x = x_2 - x_1$$

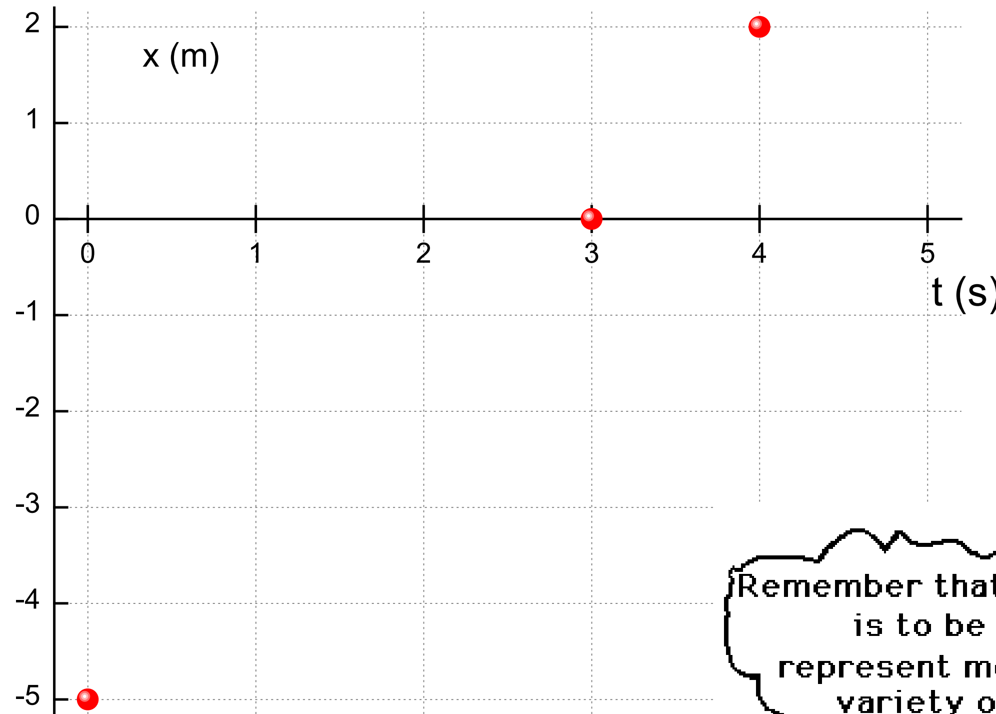
El desplazamiento es independiente del sistema de referencia

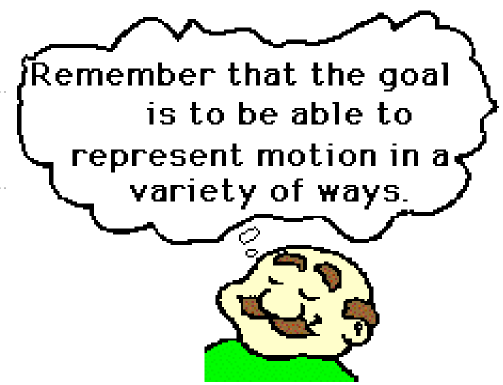
Y el tiempo... ???



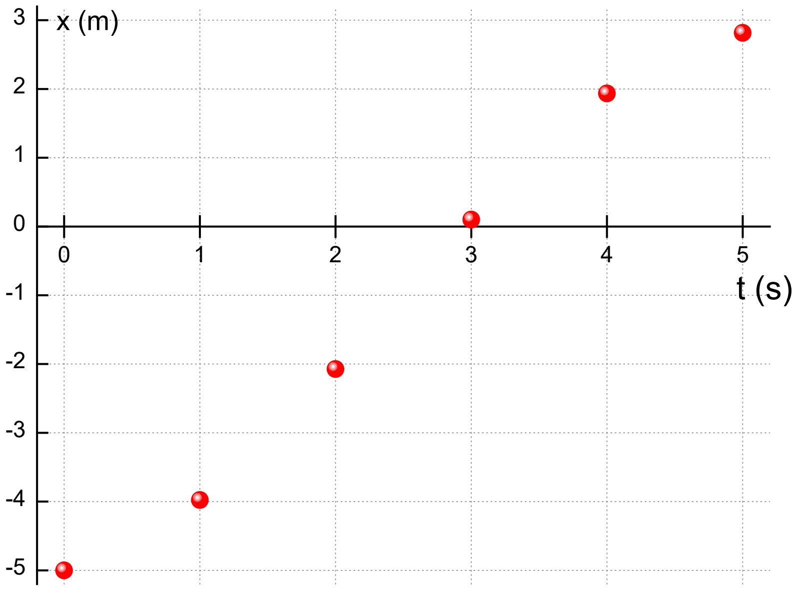
Otras formas

t (s)	x (m)
0	-5
3	0
4	+2

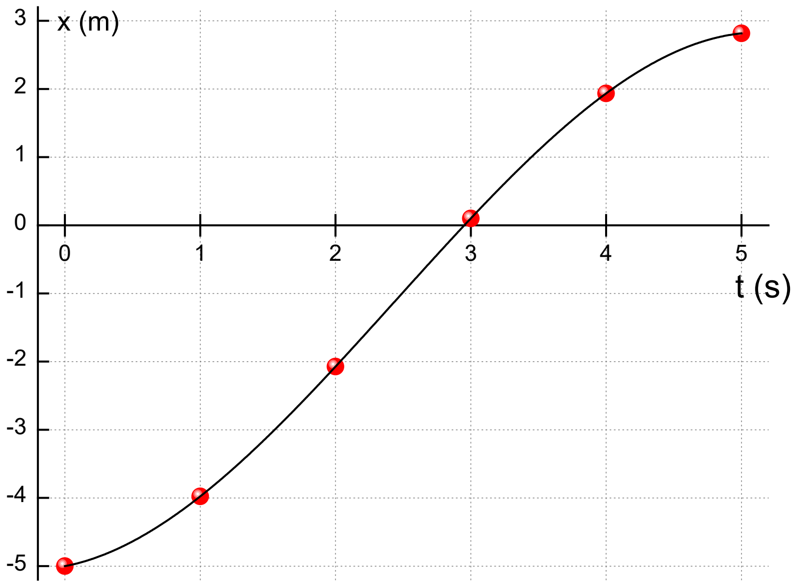


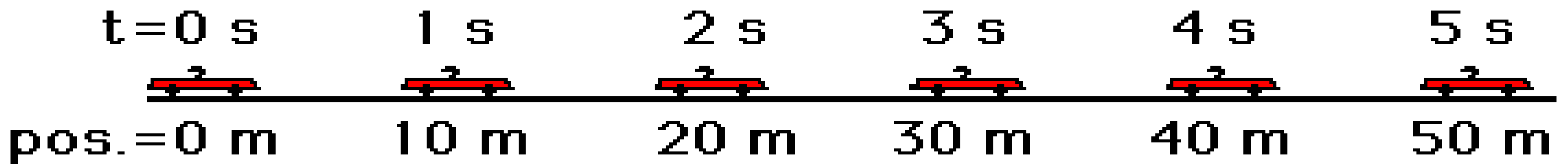


t (s)	*x (m)*
0	-5.00159
1	-3.97778
2	-2.07301
3	0.10162
4	1.93501
5	2.81606

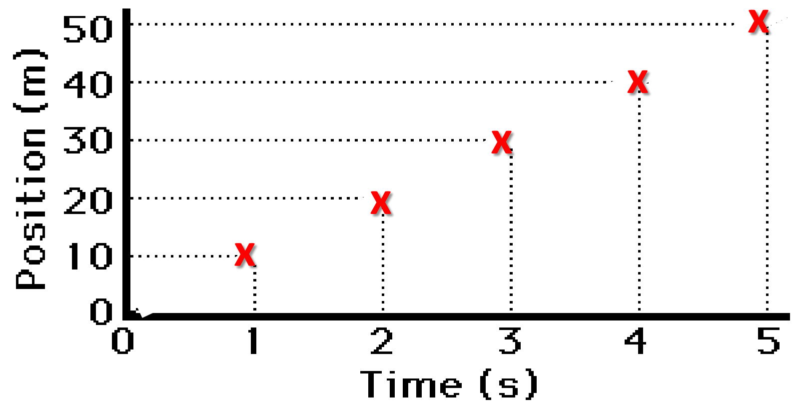


$$y = B_0 + B_1 x + B_2 x^2 + B_3 x^3$$





t (s)	*x (m)*
0	0
1	10
2	20
3	30
4	40
5	50



Velocidad

$$v_{méd} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Velocidad

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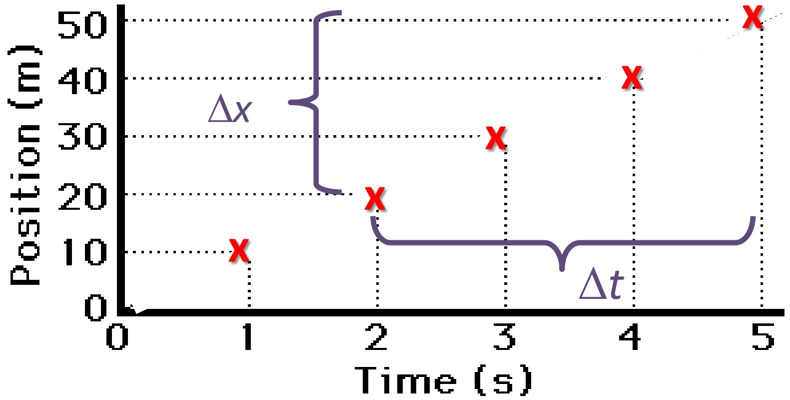
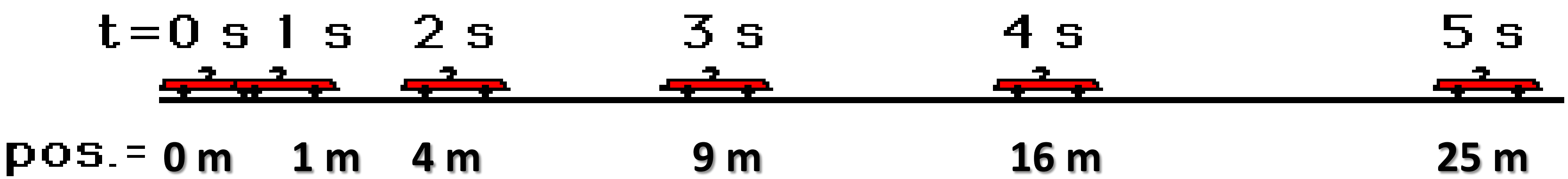
t (s)	x (m)
0	0
1	10
2	20
3	30
4	40
5	50



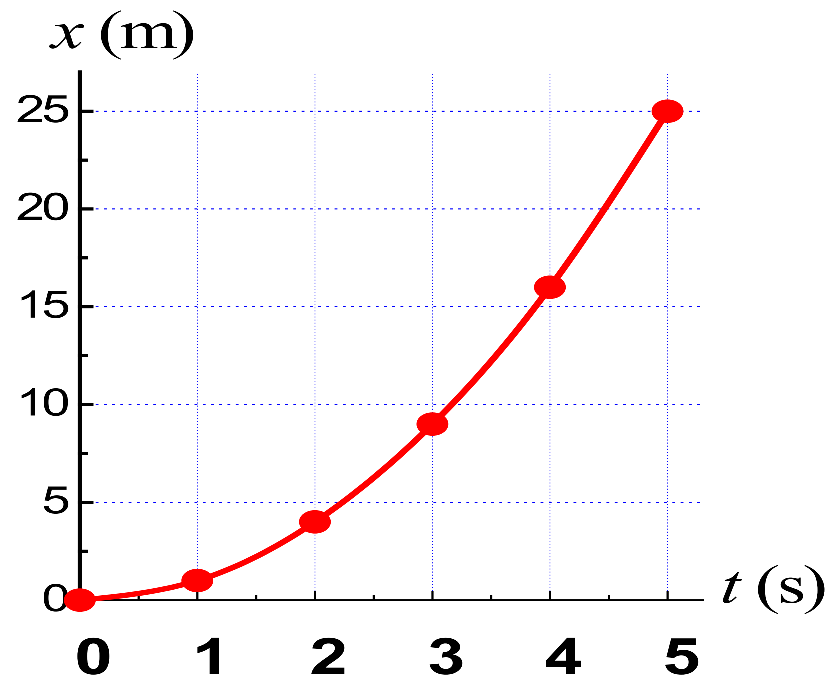
$$v_{med} = \frac{(10-0)}{(1-0)}\frac{m}{s} = 10\ m/s$$

$$v_{med} = \frac{(50-20)}{(5-2)}\frac{m}{s} = 10\ m/s$$

$$v_{med} = \frac{(40-30)}{(4-3)}\frac{m}{s} = 10\ m/s$$



t (s)	*x (m)*
0	0
1	1
2	4
3	9
4	16
5	25

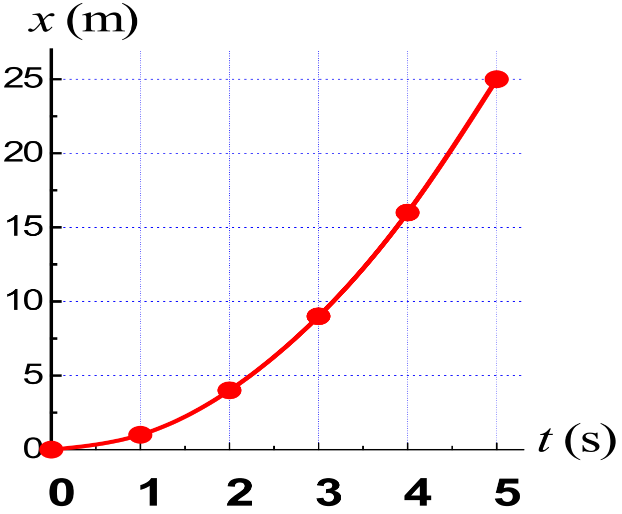


$$v_{méd} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Velocidad

$$v_{méd} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

t (s)	x (m)
0	0
1	1
2	4
3	9
4	16
5	25



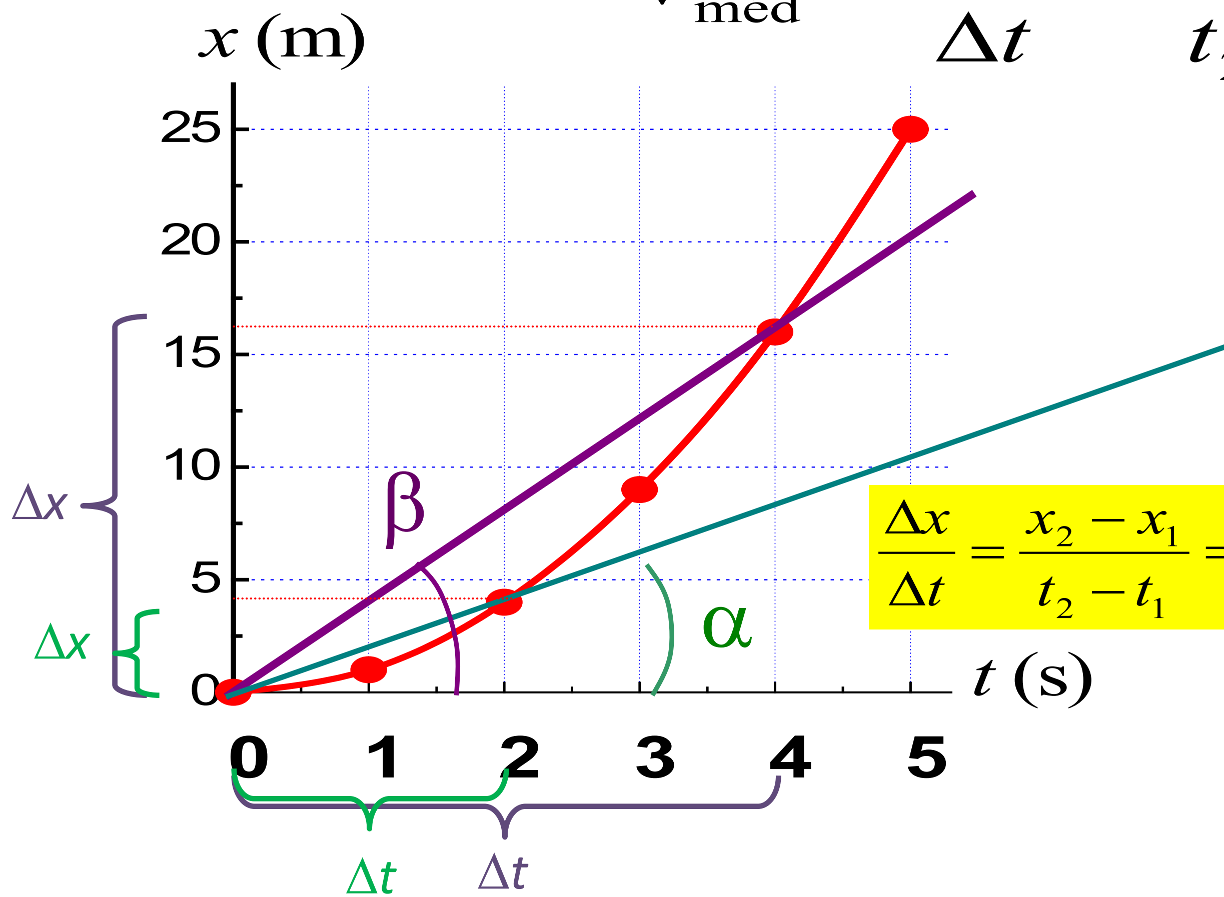
$$v_{med} = \frac{(1-0)}{(1-0)}\frac{m}{s} = 1\ m/s$$

$$v_{med} = \frac{(9-1)}{(3-1)}\frac{m}{s} = 4\ m/s$$

$$v_{med} = \frac{(25-16)}{(5-4)}\frac{m}{s} = 9\ m/s$$

Velocidad

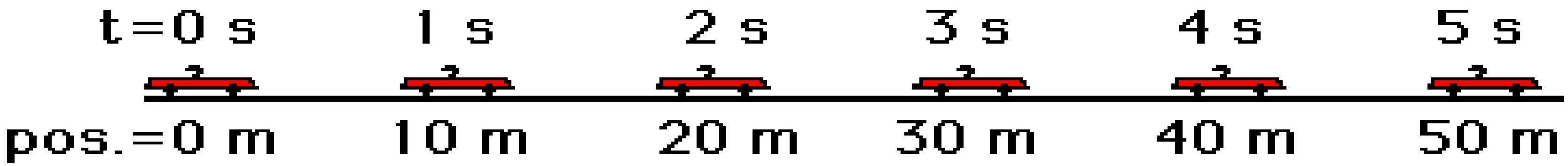
$$v_{méd} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

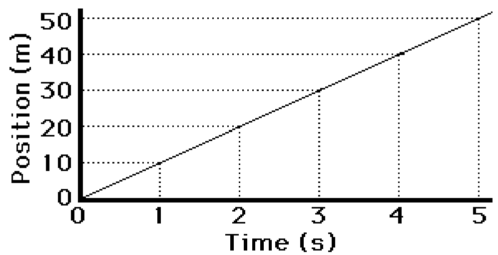


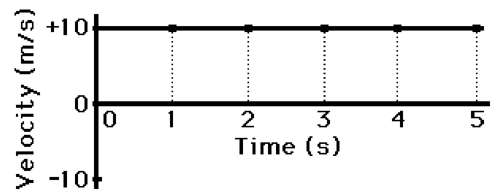
$$\frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \tan\alpha \text{ (ou } \tan\beta)$$

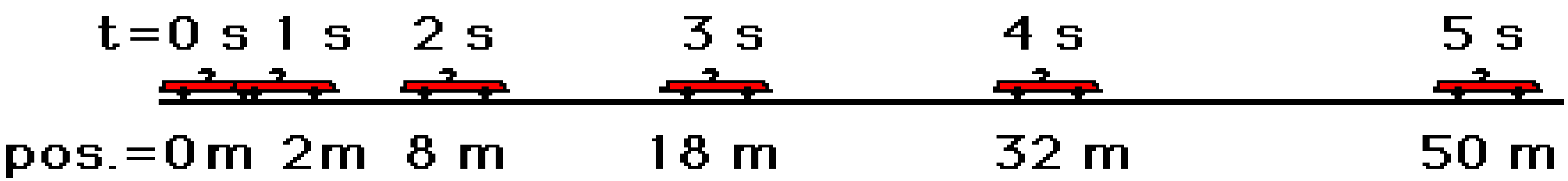
Es evidente que $\tan\alpha < tan\beta$!!!

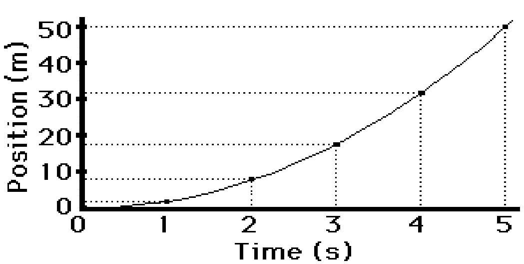
Posición, velocidad, aceleración

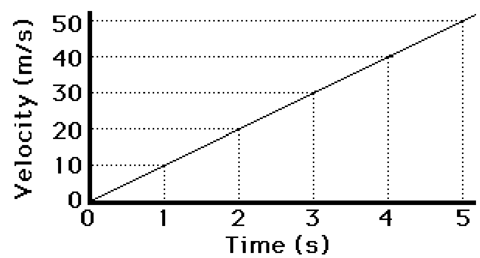


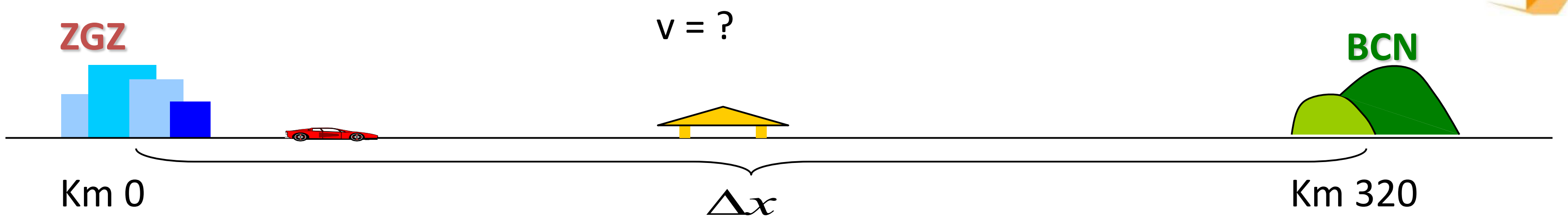












v(*t*) tiene infinitos valores!

Es una función de *t*

Es teórica

Velocidad Media

$$v_{\text{méd}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Velocidad Instantánea

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Velocidad Media

$$\mathrm{v}_{\text{méd}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Velocidad Instantánea ⇒

$$\mathrm{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Aceleración Media

$$a_{\text{méd}} = \frac{\Delta \mathrm{v}}{\Delta t} = \frac{\mathrm{v}_2 - \mathrm{v}_1}{t_2 - t_1}$$

Es 'teórica'

Aceleración Instantánea ➡

$$a(t) = \lim_{\Delta t \to 0} \frac{\Delta \mathrm{v}}{\Delta t}$$

Aceleración Instantánea

$$\mathrm{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Velocidad Media

$$v_{med} = \frac{x_f - x_i}{t_f - t_i}$$

Es un número

y

Velocidad Instantánea

$$v(t) = \frac{dx(t)}{dt}$$

Es una función de t
('receta', infinitos números)

Aceleración Media

$$a_{med} = \frac{v_f - v_i}{t_f - t_i}$$

e

Aceleración Instantánea

$$a(t) = \frac{dv(t)}{dt}$$

$$v(t) = \frac{dx(t)}{dt}$$

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}\left[\frac{dx(t)}{dt}\right]$$

$$a(t) = \frac{d^2x(t)}{dt^2}$$

Hasta nuevo aviso

$$\vec{a} = cte$$

Aceleración Constante
(Movimento Uniformemente Acelerado)

$$a_{\text{méd}} = \frac{\Delta \text{v}}{\Delta t} = \frac{\text{v}_1 - \text{v}_0}{t_1 - t_0} = \text{constante}$$

Podemos elegir $t_0 = 0$, $\text{v}_1 = \text{v}$ e $t_1 = t$ (cualquiera), entonces

$$a_{\text{méd}} = \frac{\Delta \text{v}}{\Delta t} = \frac{\text{v} - \text{v}_0}{t}$$

$$\text{v} = \text{v}_0 + a_{med} t$$

$$\Delta x = \text{v}_{\text{méd}}\, \Delta t$$

En el mismo intervalo $t - t_0 = t - 0 = t$,
El desplazamiento es

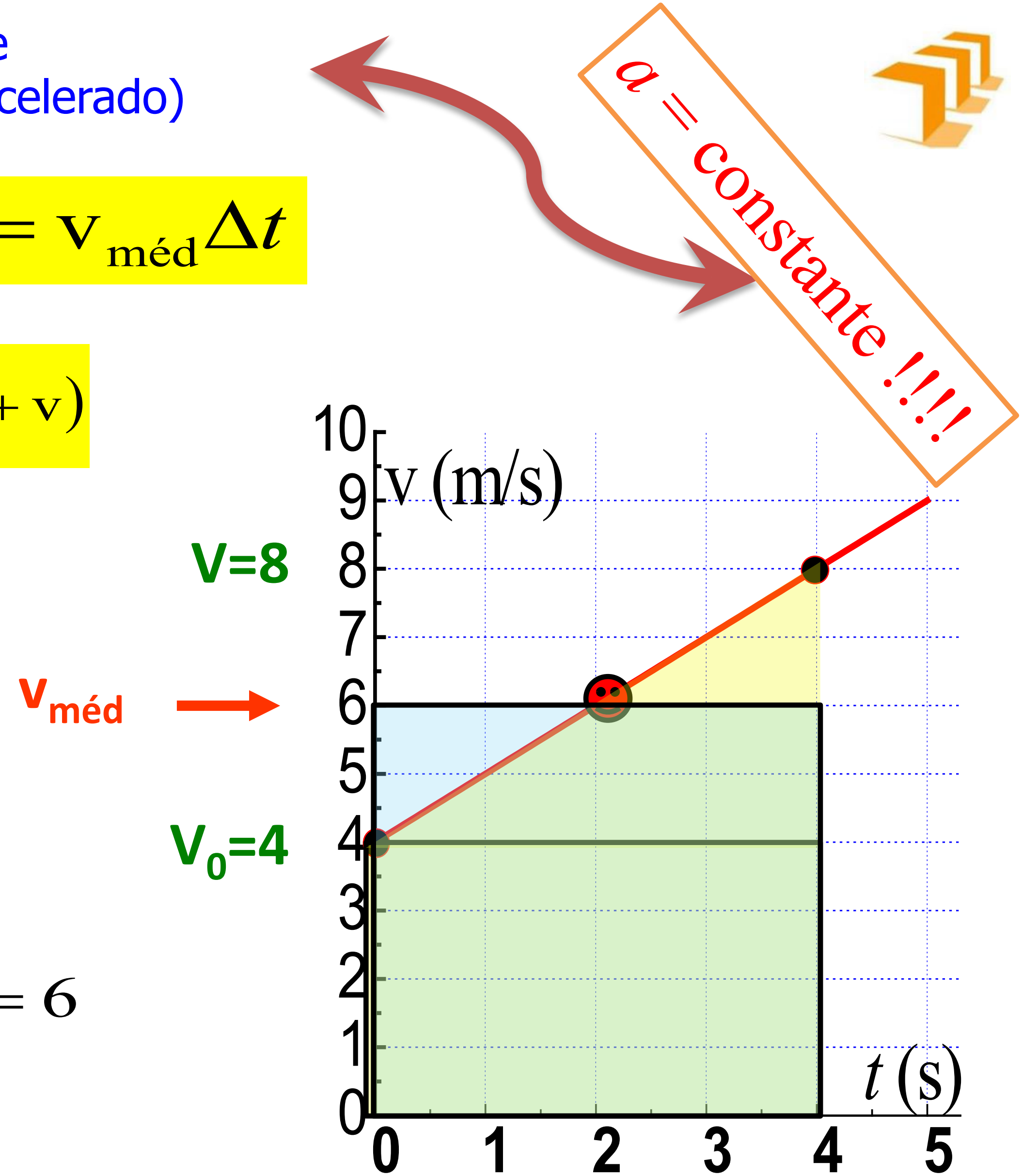
Aceleración Constante
(Movimento Uniformemente Acelerado)

Sabemos (hemos definido) que $$\mathbf{x} = \mathrm{v}_{méd}\Delta t$$

Demostraremos que $$\mathrm{v}_{méd} = \frac{1}{2}(\mathrm{v}_0 + \mathrm{v})$$

$$\mathrm{v}_{méd} = \frac{\mathrm{v} - \mathrm{v}_0}{2} + \mathrm{v}_0 = \frac{\mathrm{v} + \mathrm{v}_0}{2}$$

$$\mathrm{v}_{méd} = \frac{8-4}{2} + 4 = \frac{8+4}{2} = 6$$



Entonces:

$$\Delta x = \mathrm{v}_{\text{méd}}\ \Delta t = \mathrm{v}_{\text{méd}}\ t = \frac{1}{2}(\mathrm{v}_0 + \mathrm{v})\ t$$

Recordando que

$$\mathrm{v} = \mathrm{v}_0 + at$$

Tenemos

$$\Delta x = \frac{1}{2}(\mathrm{v}_0 + \mathrm{v})\ t = \frac{1}{2}(\mathrm{v}_0 + \mathrm{v}_0 + at)\ t$$

$$\Rightarrow x - x_0 = \mathrm{v}_0 t + \frac{1}{2}at^2$$

O bien

$$x = x_0 + \mathrm{v}_0 t + \frac{1}{2}at^2$$

$$\mathrm{v}_{\text{méd}} = \frac{1}{2}(\mathrm{v}_0 + \mathrm{v}) = \frac{x - x_0}{t}$$

$$\mathrm{v} = \mathrm{v}_0 + at$$

$$\frac{2(x - x_0)}{\mathrm{v}_0 + \mathrm{v}} = \frac{\mathrm{v} - \mathrm{v}_0}{a} \Rightarrow 2a\Delta x = \mathrm{v}^2 - \mathrm{v}_0{}^2$$

$$\mathrm{v}^2 = \mathrm{v}_0{}^2 + 2a\Delta x$$

Aceleración Constante (Movimento Uniformemente Acelerado)

En 1 dimensión tenemos:

$$x = x_0 + \mathrm{v}_0 t + \frac{1}{2} a t^2$$

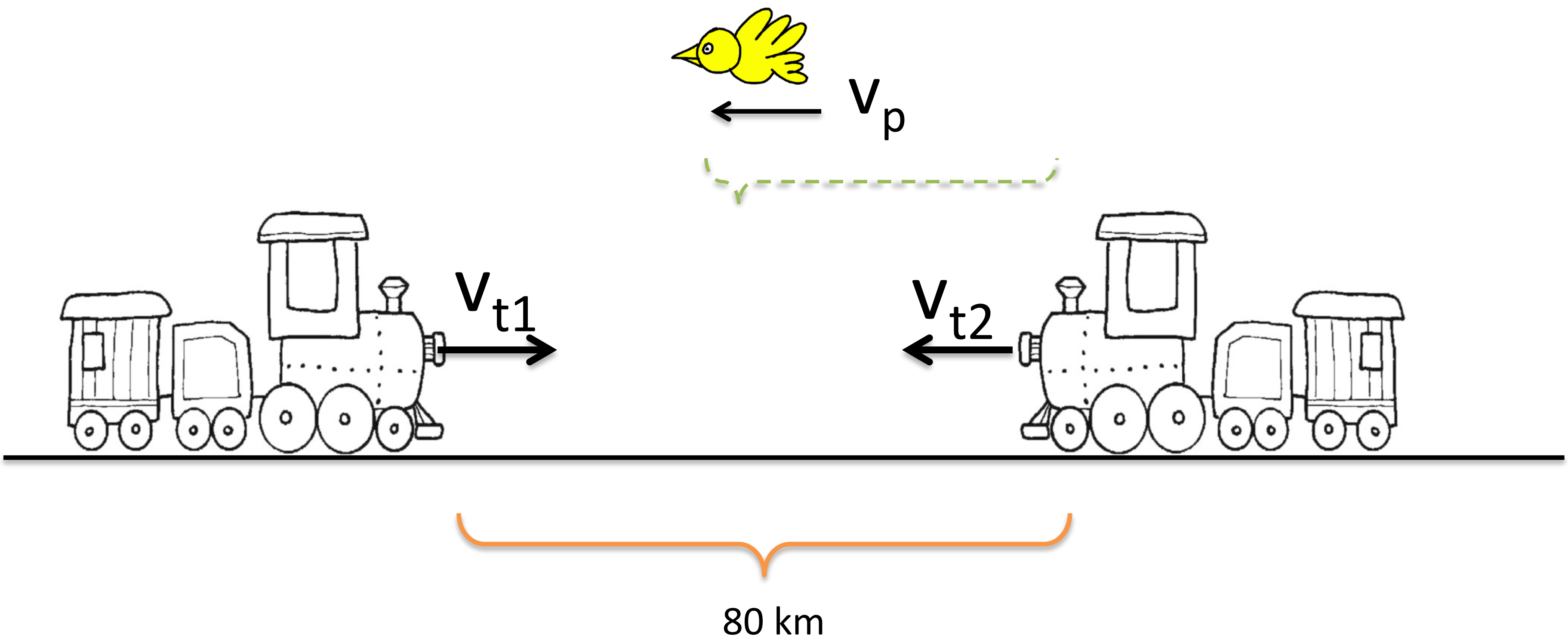
$$\mathrm{v} = \mathrm{v}_0 + a t$$

$$\mathrm{v}^2 = \mathrm{v}_0{}^2 + 2 a \Delta x$$

http://ngsir.netfirms.com/englishhtm/Kinematics.htm

1 Dimensión

Dos trenes separados 80 km se aproximan uno al otro por vías paralelas moviéndose a 10 km/h un pájaro vuela de un tren a otro en el espacio que los separa hasta que se cruzan ¿Cual es la distancia total recorrida por el pájaro, **s**, si este vuela a 20km/h?



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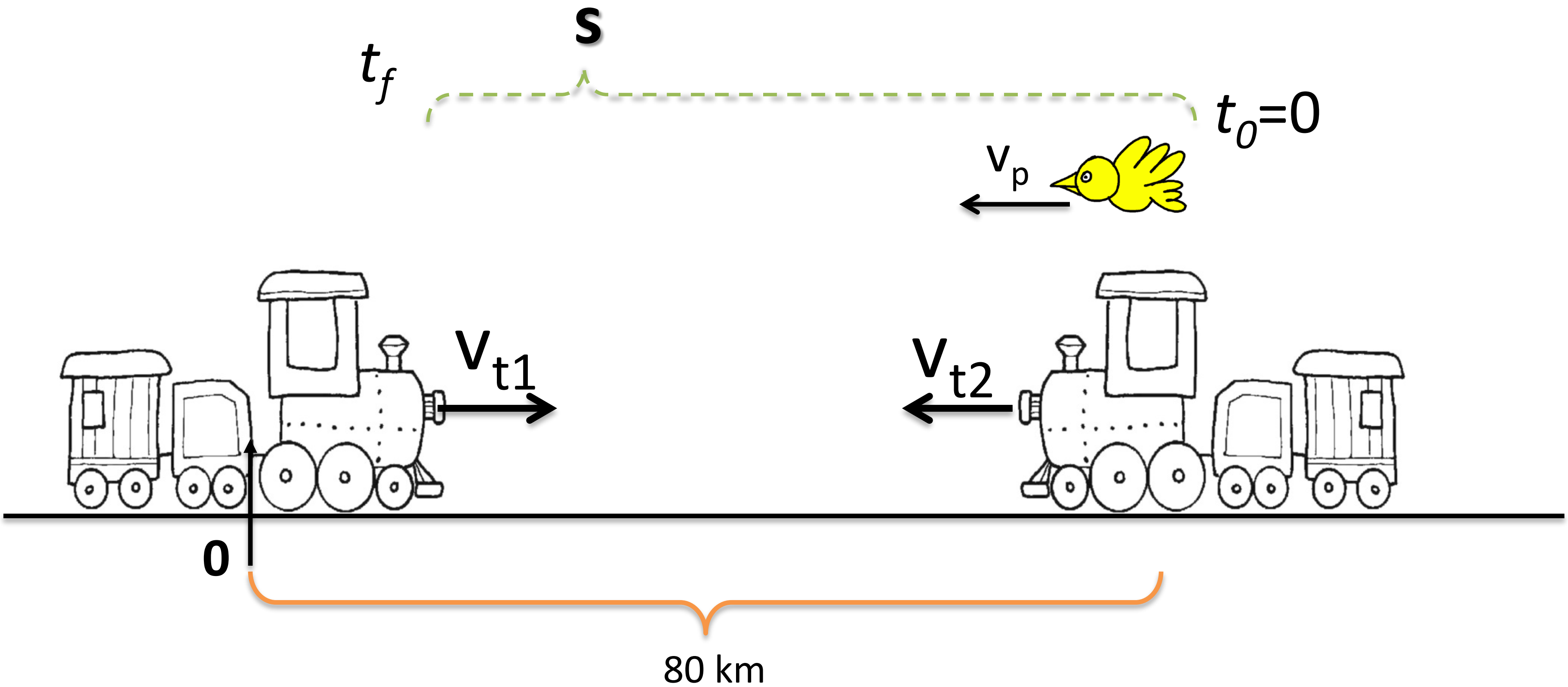

$$x_{f1} = x_{01} + v_{01}t \qquad x_{01} = 0\ km \qquad v_{01} = 10\ km/h$$

$$x_{f2} = x_{02} + v_{02}t \qquad x_{02} = 80\ km \qquad v_{02} = -10\ km/h$$

$$\begin{cases} x_{f1} = 0 + 10\ t_f \\ x_{f2} = 80 - 10\ t_f \end{cases} \qquad x_{f1} = x_{f2} \qquad 0 + 10\ t_f = 80 - 10\ t_f$$

$$t_f = 4\ h$$

$$x_{fp} = 80\ km + v_p t_f = 80\ km - 20\frac{km}{h} \times 4h = 0$$

$$S = \Delta x = x_{fp} - x_{0p}$$

$$x_{fp} - x_{0p} = 0 - 80 = -80\ km$$

$$distancia\ recorrida = \left|x_{fp} - x_{0p}\right| = 80\ km$$

FIN