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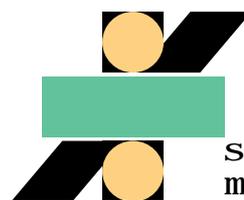
2005

“garcía de galdeano”

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n. 17



seminario  
matemático

garcía de galdeano

Universidad de Zaragoza

# ABNORMAL, PRONORMAL CONTRANORMAL AND CARTER SUBGROUPS IN SOME INFINITE GROUPS <sup>1</sup>

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## 1 Introduction

Abnormal and pronormal subgroups have appeared in a natural way in the course of investigations of some important subgroups of finite (soluble) group such as Sylow subgroups, Hall subgroups, system normalizers, and Carter subgroups. Let  $H$  be a subgroup of a group  $G$ . We recall that

1.  $H$  is *abnormal* in  $G$  if  $g \in \langle H, H^g \rangle$  for each element  $g \in G$ .
2.  $H$  is *pronormal* in  $G$  if for each element  $g \in G$ ,  $H$  and  $H^g$  are conjugate in  $\langle H, H^g \rangle$ .

Abnormal subgroups were introduced by P. Hall in his paper [HP], whereas the term *abnormal subgroup* comes from R. Carter [CR]. These subgroups and their generalizations have shown to be very useful and efficient in the finite of finite groups. Therefore, it appears to be very natural to study such fruitful concepts in infinite groups. However, it is worth to note that in some classes of infinite groups the mentioned subgroups have properties that they cannot possess in the finite case. For example, it is well-known that every finite  $p$ -group has no proper abnormal subgroups. By his way, A.Yu. Olshanskii has constructed a series of impressive examples of infinite finitely generated  $p$ -groups, which contain a lot of abnormal subgroups. Concretely, for a enough large prime  $p$ , he has constructed an infinite  $p$ -group  $G$  whose proper subgroups have prime order  $p$  ([OA, Theorem 28.1]). In particular, every proper non-identity subgroup of  $G$  is maximal and, being non-normal, is abnormal. This is very common in the infinite case, when we note very often that the situation is quite different of the situation in the corresponding finite case. Therefore, the first task is to find classes of infinite groups in which the same concept is effective at the same level as in the finite case. It is natural to start to focus our search on infinite groups that are near to finite groups; that is, infinite groups with finiteness conditions.

The aim of this survey is to discuss the main achievements obtained in the extension of some fundamental results concerning topics related to these subgroups on some classes of infinite groups. Obviously, we are unable to indicate a universal class in which such a transfer is possible. Actually, it is very usual that such a transfer can be carried out in some different but weakly related classes of infinite groups. As we will show below, in each particular case, the election of the class is not only a subjective choice. It becomes the more convenient and natural way to do so.

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<sup>1</sup>Supported by Proyecto MTM2004-04842 of Dirección General de Investigación MEC (Spain)

## 2 Abnormal and Pronormal Subgroups in Infinite Groups

We begin with the extension of the characterizations of abnormal and pronormal subgroups, which have been obtained for finite soluble groups a long time ago. It is obvious that abnormal subgroups are in a certain sense some kind of antipodes of normal subgroups. Thus, in finite soluble groups abnormality is tightly connected to self-normalizing. For example, D. Taunt has shown that *a subgroup  $H$  of a finite soluble group  $G$  is abnormal if and only if every intermediate subgroup for  $H$  coincides with its normalizer in  $G$* , that is it is self-normalizing (see, for example, [RD5, 9.2.11]). Remind that a subgroup  $S$  is said to be an *intermediate subgroup for  $H$*  if  $H \leq S$ . D. The following theorem extends this result to radical groups.

**Theorem 2.1** (*L.A. Kurdachenko, I.Ya. Subbotin [KSu2]*) *Let  $G$  be a radical group and let  $H$  be a subgroup of  $G$ . Then  $H$  is abnormal in  $G$  if and only if every intermediate subgroup for  $H$  is self-normalizing.*

The following results are straightforward consequences of above Theorem.

**Corollary 2.2** (*F. de Giovanni, G. Vincenzi [GV]*) *Let  $G$  be a hyperabelian group and let  $H$  be a subgroup of  $G$ . Then  $H$  is abnormal in  $G$  if and only if every intermediate subgroup for  $H$  is self-normalizing.*

**Corollary 2.3** *Let  $G$  be a soluble group and let  $H$  be a subgroup of  $G$ . Then  $H$  is abnormal in  $G$  if and only if every intermediate subgroup for  $H$  is self-normalizing.*

As normality, abnormality is not transitive property (to see this, it suffices to look the symmetric group  $S_4$ ). The groups (finite and infinite), in which normality is transitive, are studied well enough. However, the case of the groups with the transitivity of abnormality is very different. Actually, one cannot expect to find in the literature a detailed description here similar to the description of groups with transitive normality. However, in a finite metanilpotent group, abnormality is transitive ([R]). For infinite groups the most general result known up now is the following theorem.

**Theorem 2.4** (*L.A. Kurdachenko and I.Ya. Subbotin [KSu2]*) *Let  $G$  be a group and suppose that  $A$  is a normal subgroup of  $G$  such that  $G/A$  has no proper abnormal subgroups. If  $A$  satisfies the normalizer condition, then abnormality is transitive in  $G$ .*

We recall that a group  $G$  is said to be a group with *the normalizer condition* or  $G$  is an  $N$ -group, if  $H \neq N_G(H)$  for every proper subgroup  $H$ . Note that the class of  $N$ -groups is a proper subclass of the class of locally nilpotent groups.

We also recall that a subgroup  $H$  of a group  $G$  is said to have *the Frattini property* if given two intermediate subgroup  $K, L$  for  $H$  such that  $K$  is normal in  $L$  then  $L \leq N_G(H)K$  (in this case it is also said that  $H$  is *weakly pronormal* in  $G$ ). T.A. Peng in his paper [PT1] have characterized pronormal subgroups in finite soluble groups. He has proved that *a subgroup  $H$  of a finite soluble group  $G$  is pronormal if and only if  $H$  is weakly pronormal*.

Let  $\mathfrak{X}$  be a class of groups. A group  $G$  is said to be a *hyper- $\mathfrak{X}$ -group* if  $G$  has an ascending series of normal subgroups whose factors are  $\mathfrak{X}$ -groups. Peng's characterization of pronormal subgroups is extended in the following way.

**Theorem 2.5** (*L.A. Kurdachenko, J. Otal, I.Ya. Subbotin [KOS2]*) *Let  $G$  be a hyper- $N$ -group. Then a subgroup  $H$  of  $G$  is pronormal in  $G$  if and only if  $H$  is weakly pronormal in  $G$ .*

This result has two immediate corollaries.

**Corollary 2.6** (*F. de Giovanni, G. Vincenzi [GV]*) *Let  $G$  be a hyperabelian group and let  $H$  be a subgroup of  $G$ . Then  $H$  is pronormal in  $G$  if and only if  $H$  is weakly pronormal in  $G$ .*

**Corollary 2.7** *Let  $G$  be a soluble group and let  $H$  be a subgroup of  $G$ . Then  $H$  is pronormal in  $G$  if and only if  $H$  is weakly pronormal in  $G$ .*

T.A. Peng [PT2] has also shown that the finite soluble groups whose subgroups are all pronormal are exactly the finite soluble groups whose subgroups satisfy the transitivity for normality. Extending this result to infinite groups N.F. Kuzennyi and I.Ya. Subbotin proved the following result.

**Theorem 2.8** (N.F. Kuzennyi, I.Ya. Subbotin [KuS2]) *Suppose that  $G$  is a locally soluble group or a periodic locally graded group. Then the following conditions are equivalent.*

1. *Every cyclic subgroup of  $G$  are pronormal in  $G$ .*
2.  *$G$  is a soluble group in which all subgroups are groups with transitivity of normality.*

N.F. Kuzennyi and I. Ya. Subbotin also described completely *non-periodic locally soluble groups and periodic locally graded group in which all subgroups are pronormal* ([KuS1]), *locally graded periodic groups in which all primary subgroups are pronormal* ([KuS4]), and *infinite locally soluble groups in which all infinite subgroups are pronormal* ([KuS2]). They also have shown that in the infinite case, *the class of groups with all pronormal subgroups is a proper subclass of the class of the groups with the transitivity of normality; and moreover it is also a proper subclass of the class of groups in which all primary subgroups are pronormal.* However *the pronormality condition for all subgroups could be weakened to the pronormality for abelian subgroups* (N.F. Kuzennyi, I. Ya. Subbotin [KuS5]).

### 3 Abnormal, Pronormal, Contranormal Subgroups and Generalized Nilpotency

It is widely known the following characterization of a finite nilpotent group that are tightly connected to abnormal, pronormal and contranormal subgroups. *If  $G$  is a finite group, then the following properties are equivalent:*

1.  *$G$  is nilpotent;*
2.  *$G$  has no proper abnormal subgroups.*
3. *every pronormal subgroup of  $G$  is normal;*
4.  *$G$  has no proper contranormal subgroups;*

We recall some definitions. Let  $G$  be a group,  $H$  a subgroup of  $G$  and  $X$  a subset of  $G$ . Put

$$H^X = \langle h^x = x^{-1}hx \mid h \in H, x \in X \rangle.$$

In particular,  $H^G$  (the normal closure of  $H$  in  $G$ ) is the smallest normal subgroup of  $G$  containing  $H$ . A subgroup  $H$  is called a *contranormal subgroup* of  $G$  if  $H^G = G$  (see [R]).

Starting from the normal closure of  $H$ , we construct *the normal closure series of  $H$  in  $G$*

$$H^G = H_0 \geq H_1 \geq H_\alpha \geq H_{\alpha+1} \geq \cdots H_\gamma$$

by the following rules:  $H_{\alpha+1} = H^{H_\alpha}$  for every ordinal  $\alpha < \gamma$  and  $H_\lambda = \bigcap_{\mu < \lambda} H_\mu$  for a limit ordinal  $\lambda$ . The term  $H_\alpha$  of this series is called *the  $\alpha$ -normal closure of  $H$  in  $G$*  and will denoted in the sequel by  $H^{G,\alpha}$ . The last term  $H_\gamma$  of this series is called *the lower normal closure of  $H$  in  $G$*  and will denoted by  $H^{G,\infty}$ . If  $G$  is finite, the subgroup  $H^{G,\infty}$  is called *the subnormal closure of  $H$  in  $G$* , because in this case  $H^{G,\infty}$  is the smallest subnormal subgroup of  $G$  containing  $H$ . Obviously, every subgroup  $H$  is contranormal in its lower normal closure.

A subgroup  $H$  is called a *descendant subgroup* of a group  $G$  if  $H^{G,\infty} = H$ . A particular case of descendant subgroups are the subnormal subgroups. A *subnormal subgroup* is exactly a descending subgroup whose normal closure series is finite.

All mentioned above characterizations of nilpotency can be extended to finitely generated hyper-(abelian-by-finite) groups (and, in particular, to soluble-by-finite groups), as a consequence of the following fundamental result due to D.J.S. Robinson [RD1]: *If  $G$  is a finitely generated hyper-(abelian-by-finite) group and every finite factor-group of  $G$  is nilpotent, then  $G$  itself is nilpotent.*

In general for infinite groups we have the following results.

**Theorem 3.1** (*N.F. Kuzenny, I.Ya. Subbotin [KuS3]*) *Let  $G$  be a locally nilpotent group. Then  $G$  has no proper abnormal subgroups and every pronormal subgroup of  $G$  is normal.*

However, in the general case, we do not know whether or not the converse of this theorem holds.

L.A. Kurdachenko, J. Otal, I.Ya. Subbotin [KOS1] introduced the following wide extension of the class of soluble minimax groups. Let  $G$  be a group,  $A$  a normal subgroup of  $G$ . We say that  $A$  satisfies the condition *Max- $G$*  (respectively *Min- $G$* ) if  $A$  satisfies the maximal (respectively minimal) condition for  $G$ -invariant subgroups. A group  $G$  is said to be a *generalized minimaxgroup*, if it has a finite series of normal subgroups

$$\langle 1 \rangle = H_0 \leq H_1 \leq \dots \leq H_n = G$$

every factor of which is abelian and either satisfies *Max- $G$*  or *Min- $G$* . Every soluble minimax group is obviously generalized minimax. However, the class of generalized minimax groups is significantly wider than the class of soluble minimax groups, as the two following easy examples show.

Let  $G = \langle a \rangle \text{wr} \langle g \rangle$ , where  $a$  has prime order  $p$  and  $g$  has infinite order. Then  $G = A \rtimes \langle g \rangle$ , where  $A$  is an infinite elementary abelian  $p$ -subgroup satisfying *Max- $\langle g \rangle$* . The other example is the following. Let  $G = A \rtimes \langle g \rangle$ , where  $A$  is an infinite elementary abelian  $p$ -subgroup,  $A = \text{Dr}_{n \in \mathbb{N}} \langle a_n \rangle$ , and  $g$  is an element of infinite order such that  $g^{-1}a_1g = a_1$  and  $g^{-1}a_{n+1}g = a_{n+1}a_n$  for all  $n \in \mathbb{N}$ . Clearly,  $A$  satisfies *Min- $\langle g \rangle$* , so that  $G$  is a generalized minimax group.

In the paper [KOS1], the following generalizations of the nilpotency criteria were obtained.

**Theorem 3.2** (*L.A. Kurdachenko, J. Otal, I.Ya. Subbotin [KOS1]*) *Let  $G$  be a generalized minimax group.*

1. *if every pronormal subgroup of  $G$  is normal, then  $G$  is hypercentral; and*
2. *if  $G$  has no proper abnormal subgroups, then  $G$  is hypercentral.*

Let  $G$  be a group. Then the set

$$FC(G) = \{x \in G \mid x^G \text{ is finite}\}$$

it is easily seen to be a characteristic subgroup of  $G$ , which is called *the FC-center of  $G$* . Note that a group  $G$  is an *FC-group* if and only if  $G = FC(G)$ . Starting from the *FC-center*, we construct *the upper FC-central series* of a group  $G$

$$\langle 1 \rangle = C_0 \leq C_1 \leq \dots \leq C_\alpha \leq C_{\alpha+1} \leq \dots \leq C_\gamma,$$

where  $C_1 = FC(G)$ ,  $C_{\alpha+1}/C_\alpha = FC(G/C_\alpha)$  for all  $\alpha < \gamma$ , and  $FC(G/C_\gamma) = \langle 1 \rangle$ . The term  $C_\alpha$  is called *the  $\alpha$ -FC-hypercenter of  $G$* , while the last term  $C_\gamma$  of this series is called *the upper FC-hypercenter of  $G$* . If  $C_\gamma = G$ , then the group  $G$  is called *FC-hypercentral*, and, if  $\gamma$  is finite, then  $G$  is called *FC-nilpotent*.

**Theorem 3.3** (*L.A. Kurdachenko, A. Russo, G. Vincenzi [KRV]*) *Let  $G$  be a group whose proper subgroups are not abnormal. If  $n$  is finite and  $H = C_n$ ,*

1.  *$H$  is hypercentral; and*
2. *If  $C$  is a normal subgroup of  $G$  such that  $C \geq H$  and  $C/H$  is hypercentral, then  $C$  is hypercentral.*

**Theorem 3.4** (L.A. Kurdachenko, A. Russo, G. Vincenzi [KRV]) *Let  $G$  be an  $FC$ -nilpotent group. If  $G$  has no proper abnormal subgroups, then  $G$  is hypercentral.*

In passing, we mention that the above results were formerly obtained L.A. Kurdachenko and I.Ya. Subbotin [KSu1] for periodic groups.

The next results connects conditions of nilpotency with descendant subgroups.

**Theorem 3.5** (L.A. Kurdachenko and I.Ya. Subbotin [KSu1]) *Let  $G$  be a group, every subgroup of which is descendant. If  $G$  is  $FC$ -hypercentral, then  $G$  is hypercentral.*

**Theorem 3.6** (L.A. Kurdachenko, J. Otal and I.Ya. Subbotin [KOS2]) *Let  $G$  be a generalized minimax group. Then every subgroup of  $G$  is descendant if and only if  $G$  is nilpotent.*

## 4 The Existence of Carter Subgroups in Infinite Groups

In a finite group there exist many important families of subgroups having crucial influence on the structure of the group; for example Sylow and Hall subgroups, system normalizers, subgroups defined by formations, and so on. Many of them are specific for finite groups. However, with Carter subgroups the situation is quite different. In finite groups, *Carter subgroups* have been introduced by R. Carter [CR] as self-normalizing nilpotent subgroups. In this former definition one can find no specifications related to finite groups. These subgroups are known to be very tightly connected to abnormality. However, in infinite groups, the classical definition of Carter subgroups is useful in limited cases for some specific kinds of groups.

Initial classes of infinite groups having Carter subgroups have been investigated by S.E. Stonehewer. In his papers [SE1, SE2], he has been able to show that every periodic locally soluble groups having a locally nilpotent radical of finite index and locally soluble  $FC$ -groups have self-normalizing locally nilpotent subgroups. In other words, S.E. Stonehewer establishes the existence of Carter subgroups under their classical definition. However, this definition does not work any longer. A.D. Gardiner, B. Hartley and M.J. Tomkinson [GHT] define the class  $\mathcal{U}$  in the following way. A group  $G$  is said to belong to the class  $\mathcal{U}$ , if it has a finite series of normal subgroups with locally nilpotent factors and the Sylow  $\pi$ -subgroups of every subgroup of  $G$  are conjugated for each set  $\pi$  of primes. A.D. Gardiner, B. Hartley and M.J. Tomkinson defined Carter subgroups of  $\mathcal{U}$ -groups as locally nilpotent projectors and they established that every  $\mathcal{U}$ -group has Carter subgroups and two of them are conjugate. The same definition was used by M.R. Dixon [DM] while considering Carter subgroups in countable locally soluble groups with Chernikov Sylow subgroups.

We focus now on some recent results on Carter subgroups in infinite groups. In the beginning, we will deal with the class of nilpotent-by-hypercentral groups. There are several reasons to do so. In finite metanilpotent groups, Carter subgroups coincide with the system normalizers (see, for example, [RD5, 9.5.10]). By a result due to P. Hall (see, for example, [RD5, 9.2.15]), the system normalizers of a finite soluble group are precisely the minimal subabnormal subgroup. Note that every subabnormal subgroup in a finite metanilpotent group is abnormal. Consequently, we can define a Carter subgroup in a finite metanilpotent group as a minimal abnormal subgroup. Another reason for our choice is the following theorem implying that, in a nilpotent-by-hypercentral group, the abnormality is a transitive relation.

We describe now some approaches. Let  $\mathfrak{X}$  be a class of groups. A group  $G$  is said to be an *artinian-by- $\mathfrak{X}$ -group* if  $G$  has a normal subgroup  $H$  such that  $G/H$  belongs to  $\mathfrak{X}$  and  $H$  satisfies  $\text{Min-}G$ . Groups of this kind have been handled by D.J.S. Robinson [RD2, RD3, RD4, RD6, RD7] and D.I. Zaitsev [ZD1, ZD2, ZD3, ZD4, ZD5] in their series of papers dedicated to the existence of complements to the  $\mathfrak{X}$ -residual (when this is abelian) for some classes  $\mathfrak{X}$  of groups, such as hypercentral groups, locally nilpotent groups, hypercyclic groups, locally supersoluble groups, or hyperfinite groups.

The natural first step is the consideration of artinian-by-hypercentral groups whose locally nilpotent residual is nilpotent. For these groups, the following results hold.

**Theorem 4.1** ( L.A. Kurdachenko, I.Ya. Subbotin [KSu2]) *Let  $G$  be an artinian-by-hypercentral group and suppose that its locally nilpotent residual  $K$  is nilpotent. Then*

1.  $G$  has a minimal abnormal subgroup  $L$ . Moreover,  $L$  is maximal hypercentral subgroup and it includes the upper hypercenter of  $G$ . In particular,  $G = KL$ ; and
2. Two minimal abnormal subgroups of  $G$  are conjugated.

**Theorem 4.2** ( L.A. Kurdachenko, I.Ya. Subbotin [KSu2]) *Let  $G$  be an artinian-by-hypercentral group and suppose that its locally nilpotent residual  $K$  is nilpotent. Then*

1.  $G$  has a hypercentral abnormal subgroup  $L$ . Moreover,  $L$  is a maximal hypercentral subgroup and it includes the upper hypercenter of  $G$ . In particular,  $G = KL$ ; and
2. Two hypercentral abnormal subgroups of  $G$  are conjugated.

Thus, given an artinian-by-hypercentral group with a nilpotent hypercentral residual, a subgroup  $L$  is called a *Carter subgroup of a group  $G$*  if  $H$  is a hypercentral abnormal subgroup of  $G$  or, equivalently, if  $H$  is a minimal abnormal subgroup of  $G$ .

A Carter subgroup of a finite soluble group can be characterized as a covering subgroup for the formation of nilpotent groups. In the paper [KSu3] this definition was extended to the class of artinian-by-hypercentral groups with nilpotent locally nilpotent residual.

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