# The contribution of the fixational eye movements to the variability of the measured ocular aberration

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Root mean square difference between the original and estimated wavefront in the WSRF and ERF for eye A and B, and movement 1 and 2.
The contribution of the fixational eye movements to the variability of the measured ocular aberration

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ABSTRACT

The purpose of this work is to analyze the contribution of the eye movements to the variability of the Zernike coefficients measured with a Hartmann-Shack aberrometer. In order to isolate this effect we considered static aberrations tied to the eye pupil. We used several eye movements of different magnitude, both synthetic and corresponding to actual series recorded in our lab with different subjects. Our results show the relevance of the modal coupling induced by the estimation process and the benefit of correcting eye movements in order to get a better estimation of the ocular aberrations. They also show that eye movements during aberrometric measurements are an important source of wavefront variability.

KEYWORDS

Fixational Eye movements, Ocular Aberrometry, Hartmann-Shack, Wavefront estimation.

INTRODUCTION

Eye movements, voluntary and non voluntary, are a key factor in human vision. During steady state fixation the eye presents involuntary eye movements that exhibit an erratic trajectory with three main components: drifts, slow component with amplitude of 0.02°-0.15°; fast microsaccades, with 25 ms duration, amplitude of 0.22°-1.11° and frequency of 0.1-0.5 Hz; and tremors, with very low amplitude (0.001°-0.008°) but very high frequency (50-100 Hz) [Møller et.al. 2006, Abadi et.al. 2004].
Fixation characteristics and so eye movement properties depend on patient attention. Several works have shown that saccadic movements are affected by endogenous and exogenous sources that affect attention. They also state that the magnitude of this influence on fixational eye movements is highly subject dependent [Gowen et al. 2007].

During the last years a lot of efforts have been devoted to understand the sources of variability of the ocular aberrations. Tear film dynamics, retinal pulsation, and microfluctuations of accommodation are some of the sources that have been identified.[Montes-Mico et al. 2004, Kotulak et al. 1986, Zhu et al. 2004, Hofer et al. 2001]. However, the understanding of the ocular aberration dynamics is still incomplete. In our opinion there is a source of variability that should be taken into account: the ocular movement that occurs while looking at the fixational target during the measurement of the ocular aberrations. We consider this source as an extrinsic one. The reason is that the ocular movements do not affect the ocular aberration but they cause a change in the estimated wavefront coefficients obtained with respect to the wavefront sensor reference frame.

As said before, we can consider that in fact the ocular movements are not a source of variability of the real ocular aberration, but as it will be shown in this work they contribute to the variability of the measured aberration. Realize that the modal coefficients of the Zernike expansion of the ocular aberration are highly dependant on the position of the eye with respect to the wavefront sensor reference frame. This means that any aberrometer used to measure a hypothetical eye with static aberration that moves from measurement one to measurement two will provide us with different sets of modal coefficients, although the eye aberration will not really change.

In this work we analyze the contribution of fixational eye movements to the variability of the ocular wavefront aberration. We also study the benefit of correcting the effects of the eye movements on the estimated coefficients. In the first section we present a description of the computer simulation. Then, in the result sections we first analyze the contribution of the fixational eye movements to the variability and bias of the estimated Zernike coefficients. Second, we show the relation between the root mean square error of the estimated wavefront and the centre of the ocular trajectory. Besides, we show the
influence of the modal coupling induced in the estimation process on the reconstruction error. Finally, we present a discussion of the results.

**DESCRIPTION OF THE COMPUTER SIMULATION**

We simulated two hypothetical eyes with static aberration. The wavefront aberration was generated with Zernike polynomials up to order 12th (91 polynomials). In order to compute the value of the Zernike coefficients we used the next procedure. First, we measured with a Hartmann-Shack aberrometer the 35 first polynomials of the ocular aberration of two subjects. Then, we calculated the variance of each radial order from 2nd to 7th order using the following expression, \( \hat{\sigma}_n^2 = \sum_{m} \hat{a}_{n,m}^2 \), where the hat stands for estimated or experimental magnitudes. Then, an exponential function \( \sigma_n^2 = b \exp(-cn) \) was fitted to the values of the experimental variance. The decays (c) obtained for both eyes are compatible with that recorded by L. Thibos et.al., who established an exponential decrease of the coefficients variance with the radial order. [Thibos et.al. 2002]. The knowledge of the coefficients of the variance trend allows us to obtain variances values to orders beyond those experimentally available, for example up to 12th order, which are of interest when assessing the impact of modal coupling. Coefficients of the static aberration in the eye reference frame were then randomly generated for a 5 mm pupil diameter using a Gaussian statistical model with zero mean and an order depending variance given by the experimental fit: \( \sqrt{\frac{\sigma_n^2}{(n+1)}} \). The root mean square value ( rms ) of the generated coefficients from 36th to 91th polynomial of synthetic eye A is 4 times bigger than that of eye B (see table 1).

A Hartmann-Shack wavefront sensor was simulated. Table 1 shows its characteristics. Measurement error was not considered in the simulation. Exact knowledge of the pupil translation was assumed when performing the estimation in the eye reference frame. Fifty sets of pupil trajectories were used in the simulation, each of them with fifty positions. Two of the trajectories were obtained experimentally in our lab.

| Table 1 |
The synthetic ocular trajectories were computed using the random walk model [Engbert et al. 2004]. Inside each trajectory every position was generated adding a random value to the previous position (see equation 1):

$$\bar{x}_k = \bar{x}_{k-1} + \Delta \bar{x}_k$$  (1)

where $\bar{x}_k$ and $\bar{x}_{k-1}$ are the coordinates of the pupil centre at the $kth$ and $kth-1$ positions, and $\Delta \bar{x}_k$ is a random Gaussian variable of zero mean and certain standard deviation (in this work we considered a value of 20 microns). Refixation of the target was included in the simulation when decentrations with respect to the first position of the sequence were bigger than 200 microns. The initial position and the refixated ones were randomly generated following a Gaussian distribution of zero mean and standard deviation of 20 microns. In figure 1 we show the 50 trajectories, some of them emphasized.

Figure 1

Estimation of the modal coefficients was done using the least-squares approach [Herrmann 1981]. We used the same vector of measurements to estimate two vectors of coefficients of different length, 36 and 66 elements respectively, in order to study the effect of modal coupling [11].

In order to obtain the modal coefficients of the wavefront expansion for each of the positions we employed a translation matrix of dimensions consistent with the size of the coefficient vector (91x91) computed as described in reference [Bará et al. 2006]. The same procedure was used to compute the translation matrix used to re-express the estimated wavefront with respect to the reference frame tied to the eye pupil, in this case the dimensions were 35x35 or 66x66 depending on the size of the estimated coefficient vector.

RESULTS

In this section we include the different results that we obtained from the numerical simulation. We show first the influence of the ocular movements on the mean and variance of the measured coefficients. Then, we focus on presenting the correlation found between the trajectory’s centre and the root mean square difference between the
original wavefront and the estimated one \((rms\) error\). Finally, we assess the benefit of
expressing the estimated wavefront with respect to a reference frame tied to the eye
pupil and its dependence on the modal coupling induced in the estimation process.

**Coefficient’s mean value and standard deviation**

In this section we study the influence of the ocular movement on the mean value and
variance of the estimated coefficients of the wavefront’s Zernike expansion. We
considered the hypothetical eye B with a static wavefront aberration and an erratic
movement (as we mentioned in the description of the simulation). We estimated 66
Zernike coefficients of the wavefront aberration expansion for the different 50 positions
of the two eye trajectories measured in our lab and represented in figure 2. Then, we
computed the mean value and the variance of the estimated Zernike coefficients.

![Figure 2](image)

Figure 2 shows the results obtained for the movement 1. We represent in this figure the
absolute value of the estimated coefficients with respect to the wavefront sensor’s and
eye’s reference frame (WSRF, ERF) in gray line and bright-gray line, respectively.
Black dots represent the WSRF mean value, and white dots the ERF mean value; in
black line the original static coefficients. A magnified section of the graph is
superimposed in order to show the bias in the estimation of the mean value in the
WSRF and ERF.

We can see in figure 3 a relevant fluctuation in the absolute value of the Zernike
coefficient induced uniquely by the eye movement, being significantly higher for the
ones estimated with respect to the WSRF. The coefficients of 9\(^{th}\) order are the same in
both reference frames due to the superior triangular form of the translation matrix (see
reference Arines et.al. 2008).

![Figure 3](image)

Figure 3

In figure 4 we show the standard deviation of the estimated coefficients with respect to
the sensor (in black) and the eye (in gray) reference frames, for movement 1.
We observe in figure 4 that the standard deviation decreases when increasing the Zernike order for the coefficients estimated in the WSRF, while for those computed with respect to the ERF the standard deviation presents a flat form.

From figure 3 and 4 we can conclude that the movement of the eye induces uncertainty and variability to the estimated coefficients. This uncertainty and variability can be reduced by estimating the modal coefficients with respect to the eye’s reference frame (ERF).

**Pupil decentration and residual rms**

In order to analyze the relation between the eye movement and the uncertainty in the coefficient estimation we represented in figure 5 the rms error ($\text{rms}_{\text{w} \rightarrow \text{w}}$) versus the mean radial decentration of each trajectory ($r_j$), defining both magnitudes by the next expressions:

$$\text{rms}_{\text{w} \rightarrow \text{w}} = \sqrt{\left\langle \sum_{i=0}^{M} (a_i - \hat{a}_i)^2 \right\rangle}$$

$$r_j = \sqrt{x_j^2 + y_j^2}$$

where $a_i$, $\hat{a}_i$ are the modal coefficients of the wavefront and the estimated ones for every pupil position, $x_j$, $y_j$ are the coordinates of the positions of the $j$-th trajectory and $\langle \rangle$ is the mean value over the positions of each trajectory. The $\text{rms}_{\text{w} \rightarrow \text{w}}$ was evaluated for 50 synthetic trajectories (with 50 position each). In figure 5a and 5b we represent the $\text{rms}_{\text{w} \rightarrow \text{w}}$ computed for the coefficients estimated in the WSRF and the ERF, respectively. We also compare the results obtained for the eyes A (solid black line) and B (dotted black line); the gray scale of the right hand side of fig. 5b corresponds with the results for eye B. These eyes present a significant difference in the magnitude of their high
order coefficients (see table 1). We can see in figure 5 the clear correlation between the $\hat{W}_\text{rms}$ and the mean radial decentration. We can also see that this dependence can be highly reduced by estimating the coefficients with respect to the ERF, achieving a nearly constant $\hat{W}_\text{rms}$ up to 80-90 microns of radial decentration. Besides, figure 5b shows a difference in the value of the $\hat{W}_\text{rms}$ computed for eye A and B of nearly one order of magnitude. This result remarks the importance of the amount of the high order aberration when evaluating the influence of the ocular movement on the $\hat{W}_\text{rms}$.

Figure 5

ERF & WSRF estimation

In this section we show a comparison of the estimation process performed in both the sensor (WSRF), and eye (ERF) reference frames. Two different hypothetic eyes with different aberration (Eye A and B, see table 1) were supposed to follow two different trajectories (see figure 2). We also estimated two different sets of coefficients in each case, one with 36 coefficients and other with 66, with the aim of analyzing the contribution of modal coupling (redistribution of the contribution of the non estimated Zernike coefficients among the estimated coefficients) [Herrmann 1981].

The parameter used for the comparison was the $\hat{W}$ error ($\hat{W}_\text{rms}$), see expression (2). The ERF coefficients were obtained by multiplying the coefficients estimated in the WSRF by a translation matrix computed using the algorithm proposed in reference [Bará et.al. 2006].

In figure 6 we show the results. In black we present the $\hat{W}_\text{rms}$ computed with the coefficients estimated in the WSRF and in gray the one obtained with those computed in the ERF. Figures 6a and 6b correspond to the eyes A and B, respectively.

Figure 6

If we compare the graphs of 6a we can observe that the $\hat{W}_\text{rms}$ depends highly on the ocular movement, being significantly inferior for movement 1 (compare each bar in the
left box of figure 6a with the same color, corresponding bar in the right box). If we compare movement 1 and 2, we observe that the second one is more decentred and wider. However, if we pay attention to fig. 6b (eye B), we see that the difference between the $rms_{\hat{W}W_{rms}}$ obtained for both movements is less significant. This means that the influence of the eye movement on the estimated coefficients depends not only on the characteristics of the movements but also on the magnitude of the high order aberration (HO) of the eye. Remember that the HO of Eye B is 4 times lower than that of Eye A.

From figure 6 we can also notice a difference in the magnitude of the $rms_{\hat{W}W_{rms}}$ obtained in the WSRF and in the ERF for each eye and trajectory (compare every black bar with its adjacent gray one). Besides, there is a significant difference in the magnitude of the $rms_{\hat{W}W_{rms}}$ between both eyes under similar conditions (compare each bar in figure 6a with the corresponding bar in figure 6b). In the case of Eye B the $rms_{\hat{W}W_{rms}}$ in the ERF is nearly negligible (see the gray bars in figure 6b). Additionally, if we compare the $rms_{\hat{W}W_{rms}}$ obtained when estimating 36 and 66 coefficients in the ERF for each eye and movement, we find again an important difference (compare the gray bars inside each box of either figure).

**DISCUSSION**

Throughout this paper we analyzed the influence of the ocular movements on the estimated coefficients. To do that, we simulated two hypothetic eyes with different static aberrations and significant difference in the magnitude of their high order aberrations, following several trajectories.

All the results presented in this paper point out the influence of the ocular fixational movements on the estimated coefficients, and thus on the statistical properties attributed to the dynamics of ocular aberrations and poblational distribution of Zernike coefficients.
Figure 3 shows the different values of the Zernike coefficients that we obtained when measuring the static wavefront following an erratic trajectory, like those of fixational eye movements. The non zero mean distribution of positions causes certain bias in the estimation of the static aberration. The better accuracy and precision achieved in the coefficients estimated with respect to the eye’s reference frame (thanks to the correction of the movement by multiplying the WSRF coefficients vector by a translation matrix [Bará et.al. 2006, Arines et.al. 2008]) suggest the importance of correcting the estimated coefficients in order to get a better understanding of the statistical properties of the dynamics of the ocular aberrations.

As we said in the previous paragraph, the non zero mean of trajectory positions causes bias in the value of the estimated coefficients. Figure 5 shows that relation. In opposition to the high correlation that can be observed between the trajectory centre and the mean value of the coefficients estimated in the WSRF (see fig.5a), we observe in fig. 5b that up to 80-90 microns the $rms$ in the ERF is almost constant. Another difference between fig. 5a and 5b that should be pointed out is the magnitude of the $rms$, one order of magnitude. This difference is significant, and much more when trying to understand the sources of variability of the ocular aberrations or know their statistical properties.

One question that remains open after reading the previous paragraph is: Why can we only correct properly the displacements smaller to 80-90 microns? Initially, we would expect to be able to obtain a good correction independently on the magnitude of the displacement. The answer to this question deals with the modal coupling mentioned in previous sections. The translation matrix can correct the displacement but not the modal coupling. Moreover, we must take into account that we apply the transformation to the estimated coefficients (that present certain amount of modal coupling, whose magnitude depends on the value of the coefficients of higher order), and not to the real ones. Figure 6 supports this explanation. The $rms$ error depends on the magnitude of the high order aberration (the one of eye A is 4 times higher than that of eye B), on the movement, on the number of modes estimated and thus on the modal coupling, and on the system of reference used to estimate the coefficients. The higher the number of modes estimated the lower the modal coupling, and as can be shown in fig. 6 the correction of the movements is better.
In conclusion, the results presented in this paper show the contribution of the fixational ocular movements to the variability and loss of precision and accuracy of the estimated modal coefficients. The consideration of this movements and their correction by expressing the wavefront coefficients with respect to a reference frame tied to the eye pupil is necessary for a better understanding of the ocular aberration and statistical properties. We think that the use of eye trackers should be considered in order to measure the eye position during the aberrometric measurement to compensate-at least partially-for this source of error.

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REFERENCES:


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<td><strong>Microlenses side</strong></td>
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<td><strong>Pupil radius</strong></td>
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<td><strong>Eye A rms(_{HO})</strong></td>
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<td><strong>Eye B rms(_{HO})</strong></td>
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Table 1: Simulation parameters
FIGURE CAPTIONS

Figure 1: 50 trajectories of ocular movements

Figure 2: Movement 1 and 2 registered during aberrometric measurements

Figure 3: Absolute value of the estimated coefficients for movement 1. i modal order. The graph shows the values obtained for each position as well as coefficient mean value.

Figure 4: Standard deviation of the coefficients estimated in the WSRF (black) and ERF (gray) for eye B.

Figure 5: Residual root mean square error, \( \text{rms}_{\text{w.r.m.s.}} \), versus mean trajectory decenteration for the coefficients obtained in the WSRF (fig.5a) and ERF (fig.5b). In solid black line the case of eye A and in dotted black line the eye B.

Figure 6: Root mean square difference between the original and estimated wavefront in the WSRF and ERF for eye A and B, and movement 1 and 2.